Chapter 8

Sensitive Control Charts

CUSUM and EWMA

- *A major disadvantage of Shewhart control charts is using only the information about the process contained in the* ***last sample observation*** *and it ignores any information given by the entire sequence of points*

- *This feature makes the Shewhart control chart relatively* ***insensitive to small process shifts***

- *Two very effective alternatives to the Shewhart control chart may be used when* ***small process shifts*** *are of interest:*
  - *Cumulative Sum (CUSUM) control chart*
  - *Exponentially Weighted Moving Average (EWMA) control chart*

CUSUM Control Chart

- *Consider the data in Table 9.1, column (a)*
- *The first 20 of these observations were drawn at ran with μ=10 and σ=1*

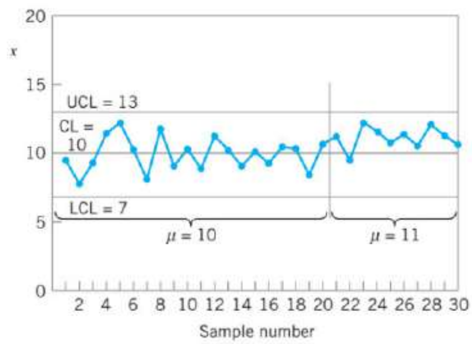
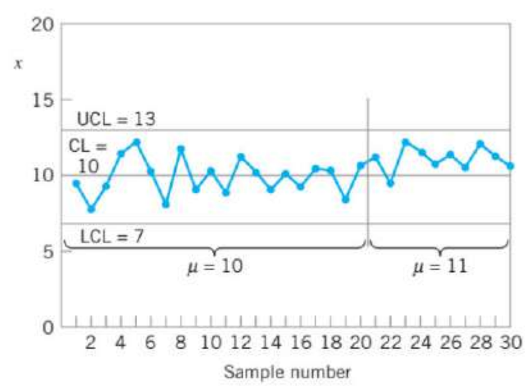


TABLE 9.1
Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

CUSUM Control Chart

- *The **reason for this failure**, of course, is the relatively **small magnitude of the shift***
- *The **Shewhart chart** for averages is very effective if the magnitude of the shift is **1.5σ or larger,** for smaller shifts, it is not as effective*
- ***CUSUM** control chart is good choice when **small shifts** are important.*



CUSUM Control Chart

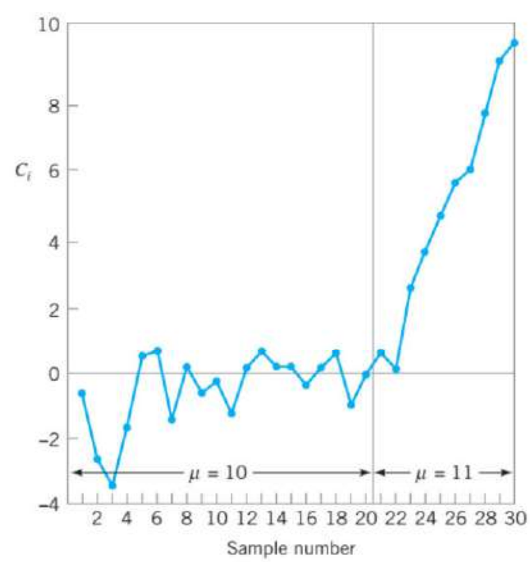
- *The CUSUM chart directly incorporates* ***all the information in the sequence*** *of sample values.*
- *It plots the cumulative sums of the deviations of the sample values from a target value.*

- *Suppose that samples of size $n \geq 1$ are collected,*
- *$\bar{x}_j$ is the average of the j^{th} sample,*
- *If μ_0 is the target for the process mean,*
- *the cumulative sum control chart is formed by plotting the quantity* $C_i = \sum_{j=1}^{i} \left(\bar{x}_j - \mu_0\right)$
- *Effective with samples of size $n = 1$.*

CUSUM Control Chart

- *Note that if the process remains* ***in control*** *at the target value μ_0, the cumulative sum equation is a* ***random walk*** *with mean zero*
- *For the previous example:*

$$C_i = \sum_{j=1}^{i} \left(x_j - 10\right) = \left(x_i - 10\right) + \sum_{j=1}^{i-1} \left(x_j - 10\right) = \left(x_i - 10\right) + C_{i-1}$$



Algorithmic CUSUM for Mean Monitoring

- *Let x_i be the i^{th} observation on the process.*
- *When the process is in control, x_i has a normal distribution with mean μ_0 and standard deviation σ*
- *We assume that either σ is known or that a reliable estimate is available*
- *We think of μ_0 as a target value for the quality characteristic x*
- *The tabular CUSUM works by accumulating derivations from μ_0 that are above target with one statistic C^+ and accumulating derivations from μ_0 that are below target with another statistic C^-.*
- *C^+ is one-sided upper CUSUMs*
- *C^- is one-sided lower CUSUMs*

Algorithmic CUSUM for Mean Monitoring

- *C^+ and C^- are computed as follows:*

The Tabular CUSUM

$$C_i^+ = \max\left[0, x_i - (\mu_0 + K) + C_{i-1}^+\right] \tag{9.2}$$

$$C_i^- = \max\left[0, (\mu_0 - K) - x_i + C_{i-1}^-\right] \tag{9.3}$$

where the starting values are $C_0^+ = C_0^- = 0$.

- *K is usually called the reference value and equals with:* $K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2}$

Algorithmic CUSUM for Mean Mc

- *If either C_i^+ or C_i^- exceeds the **decision interval H,** the process is considered to be out of control.*
- *A reasonable value for H is five times the process standard deviation σ.*

- *Example 9.1. Set up the tabular CUSUM using the data from Table 9.1.*

TABLE 9.1
Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

Algorithmic CUSUM for Mean Mc

- *Answer 9.1.*
  - *the target value is $\mu_0=10$*
  - *the process standard deviation is $\sigma=1$*
  - *the magnitude of the shift is $1.0\sigma=1.0(1.0)= 1.0$*
  - *the out-of-control value of the process mean is $\mu_1=10+1=11$*
  - *So, we should use a tabular CUSUM with K=0.5*

$$K=\frac{\delta}{2}\sigma=\frac{|\mu_1-\mu_0|}{2}$$

TABLE 9.1
Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
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27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

Algorithmic CUSUM for Mean M(

- *Answer 9.1.*
  - *the target value is* $\mu_0=10$
  - *the process standard deviation is* $\sigma=1$
  - *the magnitude of the shift is* $1.0\sigma=1.0(1.0)=1.0$*, and* $K=0.5$
  - *the out-of-control value of the process mean is* $\mu_1=10+1=11$
  - *the recommended value of the decision interval is* $H=5\sigma=5(1)=5$
  - *So, we should use a tabular CUSUM with* $K=0.5$*, and* $H=5$

TABLE 9.1
Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
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22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

Algorithmic CUSUM for M

- *Answer 9.1.*
  - *Table 9.2 presents the tabular CUSUM scheme.*
  - *The equations for* C_i^+ *and* C_i^- *are*

$$C_1^+ = \max\left[0, x_1 - 10.5 + C_0^+\right]$$

$$C_1^- = \max\left[0, 9.5 - x_1 + C_0^-\right]$$

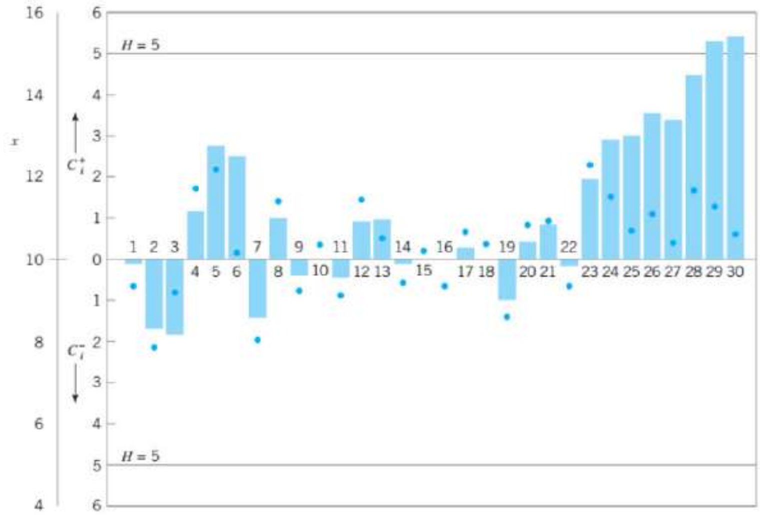
TABLE 9.2
The Tabular CUSUM for Example 9.1

		(a)			(b)		
Period i	x_i	$x_i - 10.5$	C_i^+	N^+	$9.5 - x_i$	C_i^-	N^-
1	9.45	−1.05	0	0	0.05	0.05	1
2	7.99	−2.51	0	0	1.51	1.56	2
3	9.29	−1.21	0	0	0.21	1.77	3
4	11.66	1.16	1.16	1	−2.16	0	0
5	12.16	1.66	2.82	2	−2.66	0	0
6	10.18	−0.32	2.50	3	−0.68	0	0
7	8.04	−2.46	0.04	4	1.46	1.46	1
8	11.46	0.96	1.00	5	−1.96	0	0
9	9.20	−1.3	0	0	0.30	0.30	1
10	10.34	−0.16	0	0	−0.84	0	0
11	9.03	−1.47	0	0	0.47	0.47	1
12	11.47	0.97	0.97	1	−1.97	0	0
13	10.51	0.01	0.98	2	−1.01	0	0
14	9.40	−1.10	0	0	0.10	0.10	1
15	10.08	−0.42	0	0	−0.58	0	0
16	9.37	−1.13	0	0	0.13	0.13	1
17	10.62	0.12	0.12	1	−1.12	0	0
18	10.31	−0.19	0	0	−0.81	0	0
19	8.52	−1.98	0	0	0.98	0.98	1
20	10.84	0.34	0.34	1	−1.34	0	0
21	10.90	0.40	0.74	2	−1.40	0	0
22	9.33	−1.17	0	0	0.17	0.17	1
23	12.29	1.79	1.79	1	−2.79	0	0
24	11.50	1.00	2.79	2	−2.00	0	0
25	10.60	0.10	2.89	3	−1.10	0	0
26	11.08	0.58	3.47	4	−1.58	0	0
27	10.38	−0.12	3.35	5	−0.88	0	0
28	11.62	1.12	4.47	6	−2.12	0	0
29	11.31	0.81	5.28	7	−1.81	0	0
30	10.52	0.02	5.30	8	−1.02	0	0

Algorithmic CUSUM for Mean Monitoring

- *Answer 9.1.*

– *CUSUM status chart*



Standardized CUSUM

- *Many users of the CUSUM prefer to standardize the variable x_i before performing the calculations.*
- *Let*

$$y_i = \frac{x_i - \mu_0}{\sigma}$$

The Standardized Two-Sided CUSUM

$$C_i^+ = \max\left[0, y_i - k + C_{i-1}^+\right] \quad (9.9)$$

$$C_i^- = \max\left[0, -k - y_i + C_{i-1}^-\right] \quad (9.10)$$

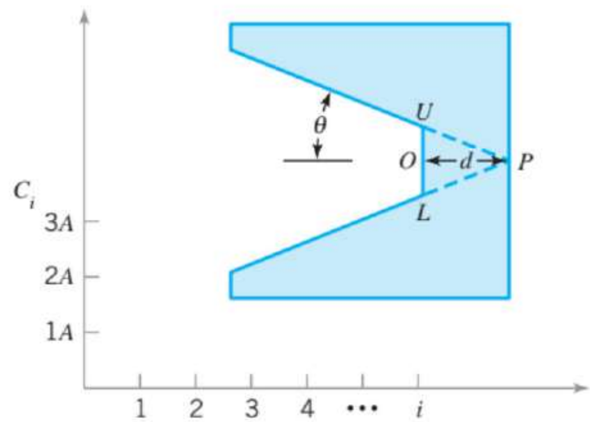
V-Mask Procedure

- *An alternative procedure to the use of a algorithmic CUSUM is the V-mask control scheme proposed by Barnard (1959)*
- *The V-mask is applied to successive values of the CUSUM statistic*

$$C_i = \sum_{j=1}^{i} y_j = y_i + C_{i-1}$$

- *where*

$$y_i = \frac{x_i - \mu_0}{\sigma}$$

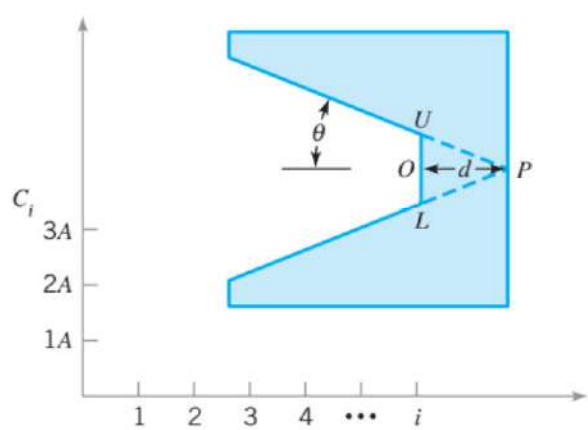


V-Mask Procedure

- *The tabular CUSUM and the V-mask scheme are equivalent if*

$$k = A\tan\theta$$

$$h = A\ d\tan(\theta) = dk$$



V-Mask Procedure

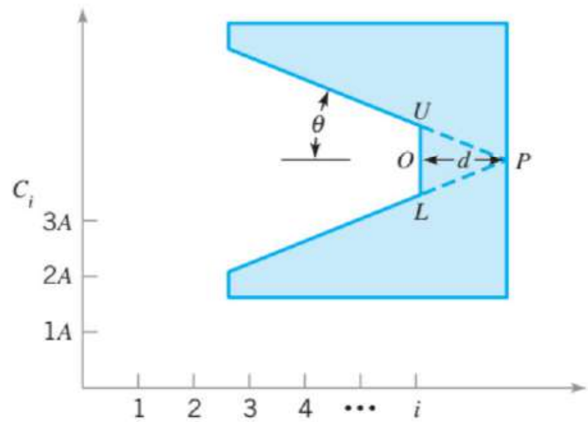
- *Example 9.1, where k=0.5 and h=5*

$$k = A\tan\theta$$

$$\frac{1}{2} = (1)\tan\theta \qquad or \qquad \theta = 26.57°$$

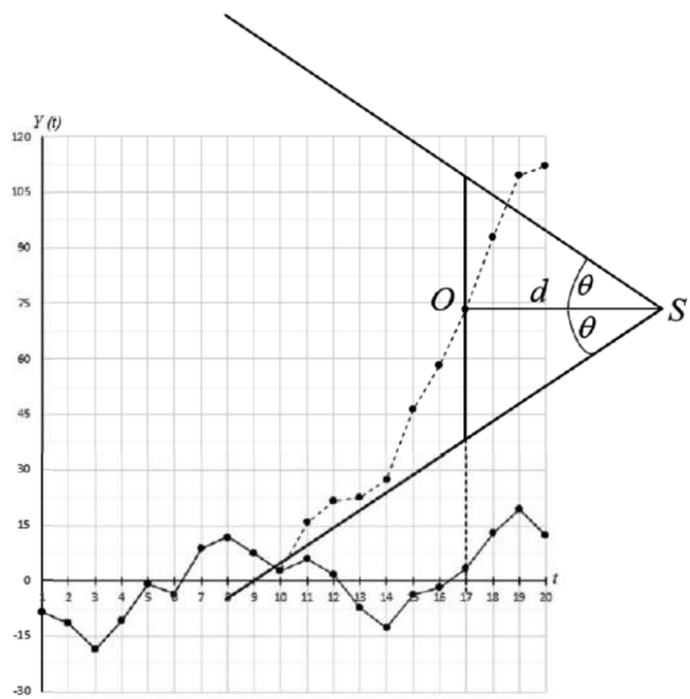
$$h = dk$$

$$5 = d\left(\frac{1}{2}\right) \qquad or \qquad d = 10$$



V-Mask Procedure

- *Example 9.1, each wing stands as a limit for the chart*



EWMA Control Chart

- *The Exponentially Weighted Moving Average (EWMA) control chart is also a good alternative to the Shewhart control chart when we are interested in detecting small shifts.*
- *The performance of the* ***EWMA*** *control chart is* ***approximately equivalent*** *to that of the* ***CUSUM*** *control chart, and in some ways it is easier to set up and operate.*
- *The EWMA control chart was introduced by Roberts (1959)*

Monitoring Average

- *The exponentially weighted moving average is defined as*

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \qquad (9.22)$$

- *where $0 < \lambda \leq 1$ is a constant and the starting value (required with the first sample at i = 1) is the process target*

$$z_0 = \mu_0$$

Monitoring Average

- *To demonstrate that the EWMA z_i is a weighted average of all previous sample means, we may substitute for z_i–1 on the right-hand side of equation 9.22 to obtain*

$$\begin{aligned} z_i &= \lambda x_i + (1-\lambda)\left[\lambda x_{i-1} + (1-\lambda) z_{i-2}\right] \\ &= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2} \end{aligned}$$

- *Continuing to substitute recursively for z_{i-j}, $j = 2, 3, \ldots, t$, we obtain*

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \tag{9.23}$$

Monitoring Average

- *The weights $\lambda(1-\lambda)^j$ decrease geometrically with the age of the sample mean*
- *Furthermore, the weights sum to unity, since*

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda\left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)}\right] = 1-(1-\lambda)^i$$

- *Because these weights decline geometrically when connected by a smooth curve, the EWMA is sometimes called a* ***geometric moving average (GMA)****.*

$$\mathrm{AM}(x_1, \ldots, x_n) = \frac{x_1 + \cdots + x_n}{n}$$

$$\mathrm{GM}(x_1, \ldots, x_n) = \sqrt[n]{|x_1 \times \cdots \times x_n|}$$

$$\mathrm{HM}(x_1, \ldots, x_n) = \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

Monitoring Average

- *If the observations x_i are independent random variables with variance σ^2, then the variance of z_i is*

$$\sigma_{z_i}^2 = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

Monitoring Average

- *The center line and control limits for the EWMA control chart are as follows:*

The EWMA Control Chart

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \qquad (9.25)$$

$$\text{Center line} = \mu_0$$

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \qquad (9.26)$$

- *the factor L is the width of the control limits*

Monitoring Average

- *The control limits will approach steady-state values given by*

$$\mathrm{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \qquad (9.27)$$

$$\mathrm{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \qquad (9.28)$$

Monitoring Average

- *Example 9.2. Set up an EWMA control chart with λ=0.10 and L=2.7 to the data in Table 9.1.*

TABLE 9.1
Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

Monitoring Average

- *Answer 9.2.*

– *the target value of the mean is* $\mu_0=10$

– *the standard deviation is* $\sigma=1$

– *the calculations for the EWMA control chart are summarized in Table 9.10*

– *the first observation,* $x_1 = 9.45$. *The first value of the EWMA is*

$$\begin{aligned} z_1 &= \lambda x_1 + (1-\lambda) z_0 \\ &= 0.1(9.45) + 0.9(10) \\ &= 9.945 \end{aligned}$$

$$\begin{aligned} z_2 &= \lambda x_2 + (1-\lambda) z_1 \\ &= 0.1(7.99) + 0.9(9.945) \\ &= 9.7495 \end{aligned}$$

Monitoring Average

- *Answer 9.2.*

TABLE 9.10
EWMA Calculations for Example 9.2

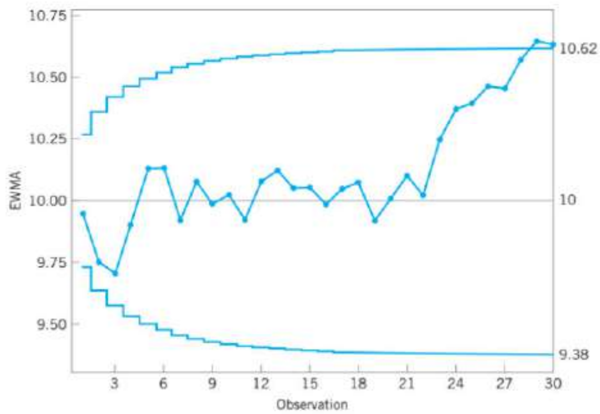
Subgroup, i	* = Beyond Limits x_i	EWMA, z_i	Subgroup, i	* = Beyond Limits x_i	EWMA, z_i
1	9.45	9.945	16	9.37	9.98426
2	7.99	9.7495	17	10.62	10.0478
3	9.29	9.70355	18	10.31	10.074
4	11.66	9.8992	19	8.52	9.91864
5	12.16	10.1253	20	10.84	10.0108
6	10.18	10.1307	21	10.9	10.0997
7	8.04	9.92167	22	9.33	10.0227
8	11.46	10.0755	23	12.29	10.2495
9	9.2	9.98796	24	11.5	10.3745
10	10.34	10.0232	25	10.6	10.3971
11	9.03	9.92384	26	11.08	10.4654
12	11.47	10.0785	27	10.38	10.4568
13	10.51	10.1216	28	11.62	10.5731
14	9.4	10.0495	29	11.31	10.6468*
15	10.08	10.0525	30	10.52	10.6341*

Monitoring Average

- *Answer 9.2.*

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$
$$= 10 + 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]}$$
$$= 10.27$$

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$
$$= 10 - 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]}$$
$$= 9.73$$

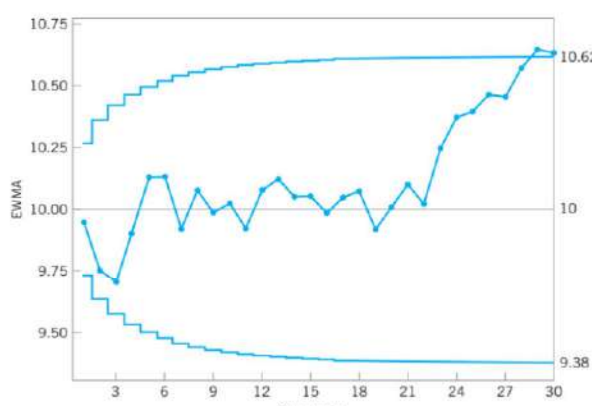


Monitoring Average

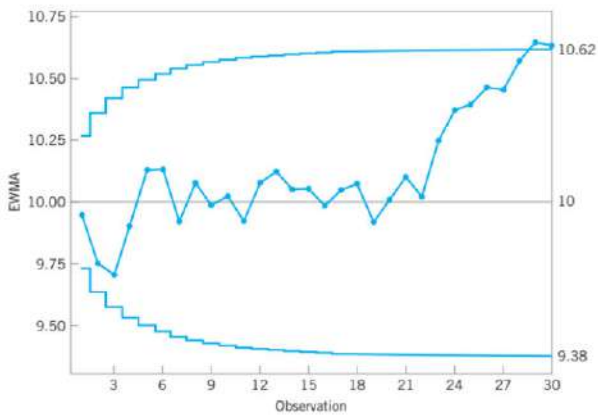
- *Answer 9.2.*

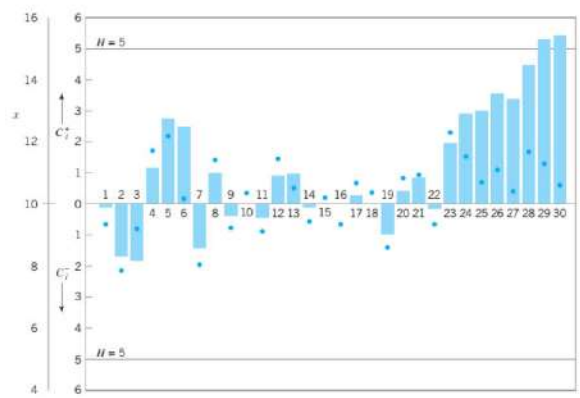
$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$
$$= 10 + 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}}$$
$$= 10.62$$

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$
$$= 10 - 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}}$$
$$= 9.38$$



EWMA vs CUSUM





Monitoring Variability

- *MacGregor and Harris (1993) discuss the use of EWMA based statistics for monitoring the process standard deviation.*

- *Let x_i be **normally distributed** with mean μ and standard deviation σ.*

- *The Exponentially Weighted Mean Square error (EWMS) is defined as*

$$S_i^2 = \lambda(x_i - \mu)^2 + (1-\lambda)S_{i-1}^2$$

Monitoring Variability

- *It can be shown that*

$$E(S_i^2) = \sigma^2$$

- *If the observations are independent and normally distributed (for large i), then:*

$$S_i^2/\sigma^2$$

has an approximate chi-square distribution with $\nu = (2 - \lambda)/\lambda$

Monitoring Variability

- *Therefore, if σ_0 represents the in-control or target value of the process standard deviation, we could plot $\sqrt{S_i^2}$ on an Exponentially Weighted Root Mean Square error (EWRMS) control chart with control limits given by*

$$\text{UCL} = \sigma_0 \sqrt{\frac{\chi^2_{v,\alpha/2}}{v}} \qquad \text{LCL} = \sigma_0 \sqrt{\frac{\chi^2_{v,1-(\alpha/2)}}{v}}$$

- *MacGregor and Harris (1993) point out that the EWMS statistic can be sensitive to shifts in both the process mean and the standard deviation.*
- *The Exponentially Weighted Moving Variance (EWMV) can be derive using the above control limits.*
- *If μ is unknown, estimate it using the ordinary EWMA z_i*

EWMA for Poisson Distribution

- *If x_i follows a Poisson distribution with parameter λ, then the basic EWMA recursion remains unchanged:*

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}$$

- *with $z_0 = \mu_0$ the control chart parameters are as follows:*

$$\text{UCL} = \mu_0 + A_U \sqrt{\frac{\lambda \mu_0}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$\text{Center line} = \mu_0$$

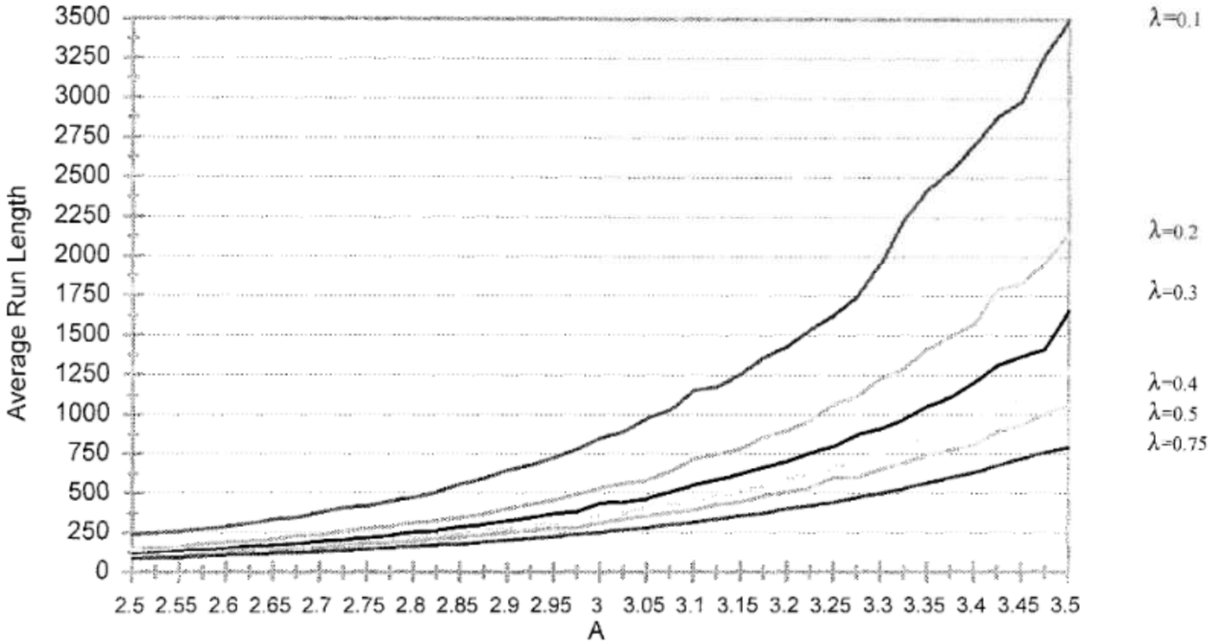
$$\text{LCL} = \mu_0 - A_L \sqrt{\frac{\lambda \mu_0}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

EWMA for Poisson Distribution

- *A_U and A_L are the upper and lower control limit factors.*

- *In many applications we would choose $A_U = A_L = A$.*

- *Borror, Champ, and Rigdon (1998) give graphs of the ARL performance of the Poisson EWMA control chart as a function of l and A and for various in-control*



Moving Average Control Chart

- *Both the CUSUM and the EWMA are* **time-weighted** control charts.
- *Suppose that individual observations have been collected, and let* x_1, x_2, ... *denote these observations.*
- *The moving average of span w at time i is defined as*

$$M_i = \frac{x_i + x_{i-1} + \cdots + x_{i-w+1}}{w} \qquad (9.37)$$

- *The variance of the moving average* M_i *is*

$$V(M_i) = \frac{1}{w^2} \sum_{j=i-w+1}^{i} V(x_j) = \frac{1}{w^2} \sum_{j=i-w+1}^{i} \sigma^2 = \frac{\sigma^2}{w} \qquad (9.38)$$

Moving Average Control Chart

- *Therefore, if μ_0 denotes the target value of the mean used as the center line of the control chart,*
- *Then the three-sigma control limits for M_i are*

$$\mathrm{UCL} = \mu_0 + \frac{3\sigma}{\sqrt{w}} \quad (9.39)$$

$$\mathrm{LCL} = \mu_0 - \frac{3\sigma}{\sqrt{w}} \quad (9.40)$$

Moving Average Control Chart

- *Example 9.3. Set up a moving average control chart for the data in Table 9.1, using w = 5.*

- *Answer 9.3.*
  - *the statistic plotted on the moving average control chart will be*

$$M_i = \frac{x_i + x_{i-1} + \cdots + x_{i-4}}{5}$$

TABLE 9.1

Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

Moving Average Control Chart

- *Answer 9.3.*
  - *the observations x_i for periods $1 \le i \le 30$ are shown in Table 9.14*
  - *The values of these moving averages are shown in Table 9.14.*

TABLE 9.14
Moving Average Chart for Example 9.3

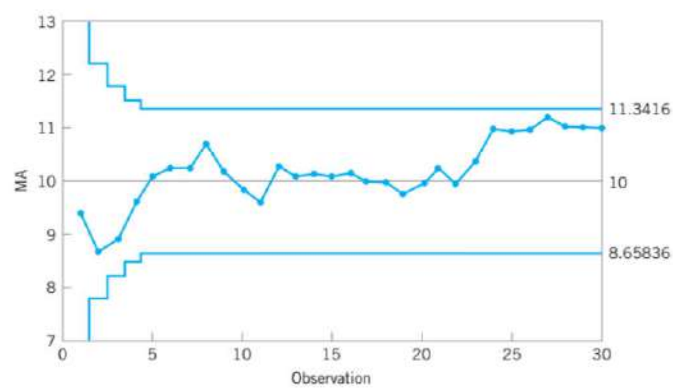
Observation, i	x_i	M_i	Observation, i	x_i	M_i
1	9.45	9.45	16	9.37	10.166
2	7.99	8.72	17	10.62	9.996
3	9.29	8.91	18	10.31	9.956
4	11.66	9.5975	19	8.52	9.78
5	12.16	10.11	20	10.84	9.932
6	10.18	10.256	21	10.9	10.238
7	8.04	10.266	22	9.33	9.98
8	11.46	10.7	23	12.29	10.376
9	9.2	10.208	24	11.5	10.972
10	10.34	9.844	25	10.6	10.924
11	9.03	9.614	26	11.08	10.96
12	11.47	10.3	27	10.38	11.17
13	10.51	10.11	28	11.62	11.036
14	9.4	10.15	29	11.31	10.998
15	10.08	10.098	30	10.52	10.982

Moving Average Control Chart

- *Answer 9.3.*
  - *The control limits for the moving average control chart may be easily obtained from equations 9.39 and 9.40.*
  - *We know that σ=1.0*

$$UCL = \mu_0 + \frac{3\sigma}{\sqrt{w}} = 10 + \frac{3(1.0)}{\sqrt{5}} = 11.34$$

$$LCL = \mu_0 - \frac{3\sigma}{\sqrt{w}} = 10 - \frac{3(1.0)}{\sqrt{5}} = 8.66$$



Modern quality definition: Minimizing the variations

- 1. Jack Welch Quote
- 2. US vs Japan car industry

General structure:

$UCL = \mu + L\sigma$

$CL = \mu$

$LCL = \mu - L\sigma$

- Shewhart Charts
  - For Variables
    - $\bar{x}$, R: for small sample size (≤ 3)
    - $\bar{x}$, S: general form
    - $\bar{x}$, S^2: when the magnitude of variability is critical
  - For Attributes
    - p-chart: deal with accept or reject with Binomial dist.
    - np-chart: counting only in a/r case with Binomial dist.
    - c-chart: when counting matters distribution is Poisson
    - u-chart: when average matters distribution is Poisson
    - g and h -chart: when the distribution is Geometric
- Sensitive Charts
  - CUSUM: detecting small shifts ($\leq 1.5\sigma$) in process mean
  - EWMA: detecting small shifts ($\leq 1.5\sigma$) in process mean
  - EWMV: detecting small shifts ($\leq 1.5\sigma$) in process mean and variance
  - MA: for time-independent controlling, not useful very small shifts