Chapter 7

Attributes Control Charts

Control Charts for Attributes

- *Many quality characteristics cannot be conveniently represented numerically.*
- *Quality characteristics of this type are called attributes; for examples:*
  - *the proportion of warped automobile engine connecting rods in a day's production*
  - *the number of nonfunctional semiconductor chips on a wafer*
  - *the number of errors or mistakes made in completing a loan application*
  - *the number of medical errors made in a hospital*

Control Charts for Attributes

- *We usually classify each item inspected as either conforming or nonconforming to the specifications on that quality characteristic*
- *The terminology* ***defective*** *or* ***non-defective****, and* ***conforming*** *or* ***nonconforming****, used to identify these two classifications of product*
- *We will discuss three widely used attributes control charts:*
  - *Control chart for fraction nonconforming, or* ***p chart***
  - *Control chart for nonconformities, or the* ***c chart***
  - *Control chart for nonconformities per unit, or the* ***u chart***

Control Chart for Fraction Nonconforming

- *The* ***fraction nonconforming*** *is defined as the ratio of the number of nonconforming items in a population to the total number of items in that population.*
- *The items may have several quality characteristics that are examined simultaneously by the inspector.*
- *If the item does not conform to standard on one or more of these characteristics, it is classified as nonconforming.*
- *The statistical principles underlying the control chart for fraction nonconforming are based on the* ***binomial distribution****.*

Control Chart for Fraction Nonconforming

- *Suppose the production process is operating in a stable manner, such that the probability that any unit will not conform to specifications is p,*
- *Also, the successive units produced are* ***independent****.*
- *Then each unit produced is a realization of a* ***Bernoulli random variable*** *with parameter* ***p****.*
- *If a random sample of* ***n*** *units of product is selected, and if* ***D*** *is the number of units of product that are* ***nonconforming****, then D has a* ***binomial distribution*** *with parameters n and p; that is,*

$$P\{D=x\}=\binom{n}{x}p^x(1-p)^{n-x} \qquad x=0,\ 1,\ \ldots,\ n$$

Control Chart for Fraction Nonconforming

- *The* ***sample fraction nonconforming*** *is defined as the ratio of the number of nonconforming units in the sample D to the sample size n—that is,*

$$\hat{p}=\frac{D}{n}$$

- *It has binomial distribution, with the mean and variance of:*

$$\mu_{\hat{p}}=p \qquad\qquad \sigma^2_{\hat{p}}=\frac{p(1-p)}{n}$$

p chart (*Fraction Nonconforming*)

- *If w is a statistic that measures a quality characteristic, and if the mean of w is μ_w and the variance of w is σ_w^2, then the general model for the Shewhart control chart is as follows:*

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line } &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned}$$

- *where L is the distance of the control limits from the center line, in multiples of the standard deviation of w.*
- *It is customary to choose L = 3*

p chart (*Fraction Nonconforming*)

- *Suppose that the true fraction nonconforming p in the production process is known or is a specified* ***standard value***.

Fraction Nonconforming Control Chart: Standard Given

$$\begin{aligned} \text{UCL} &= p + 3\sqrt{\frac{p(1-p)}{n}} \\ \text{Center line } &= p \\ \text{LCL} &= p - 3\sqrt{\frac{p(1-p)}{n}} \end{aligned} \qquad (7.6)$$

p chart (Fraction Nonconforming)

- *When the process fraction nonconforming p is **not known**,*
- *So, select m preliminary samples, each of size n, as a general rule, m should be at least 20 or 25, and n should be 4 to 6*
- *Then if there are D_i nonconforming units in sample i, we compute the fraction nonconforming in the i^{th} sample as*

$$\hat{p}_i = \frac{D_i}{n} \qquad i = 1, 2, \ldots, m$$

- *and the average of these individual sample fractions nonconforming is*

$$\overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$

p chart (Fraction Nonconforming)

- *The statistic $\overline{p}$ estimates the unknown fraction nonconforming p.*
- *The center line and control limits of the control chart for fraction nonconforming are computed as follows:*

Fraction Nonconforming Control Chart: No Standard Given

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

$$\text{Center line} = \overline{p} \qquad (7.8)$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

p chart (Fraction Nonconforming)

- *Example 7.1 Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel.*
  - *By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel.*
  - *Set up a control chart to improve the fraction of nonconforming cans produced by this machine*

p chart (Fraction Nonconforming)

TABLE 7.1
Data for Trial Control Limits, Example 7.1, Sample Size $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

p chart (Fraction Nonconforming)

- *Answer: to establish the control chart*
- *m = 30 samples of n = 50 cans*

$$\overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

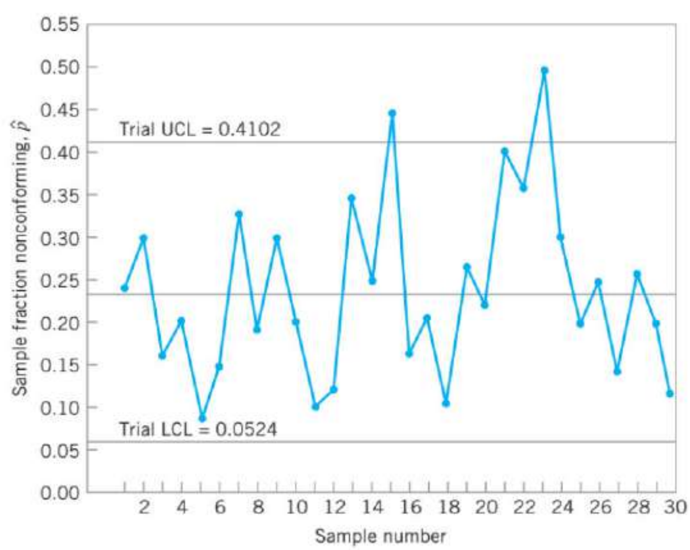
$$\begin{aligned}\overline{p} \pm 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} &= 0.2313 \pm 3\sqrt{\frac{0.2313(0.7687)}{50}} \\ &= 0.2313 \pm 3(0.0596) \\ &= 0.2313 \pm 0.1789\end{aligned}$$

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$

p chart (Fraction Nonconforming)

- *Answer:*
- *Out of control chart!*



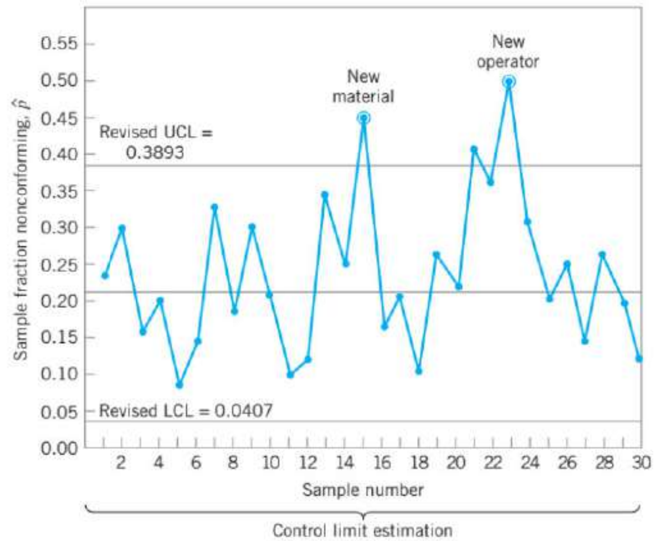
p chart (Fraction Nonconforming)

- *Answer.*
- *In control chart!*

$$\overline{p} = \frac{301}{(28)(50)} = 0.2150$$

$$\text{UCL} = 0.2150 + 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.3893$$

$$\text{LCL} = 0.2150 - 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.0407$$



p chart (Fraction Nonconforming)

- *Selection of sample size in a critical question in these charts*
- *Duncan (1986) has suggested that the sample size should be large enough that we have approximately a 50% chance of detecting a process shift of some specified amount.*
- *If δ is the magnitude of the process shift, then n must satisfy* $\delta = L\sqrt{\frac{p(1-p)}{n}}$

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p)$$

p chart (Fraction Nonconforming)

- *For example, suppose that p = 0.01, and we want the probability of detecting a shift to p = 0.05 to be 0.50.*
- *Then, p = 0.01, δ = 0.05 − 0.01 = 0.04, and if three-sigma limits are used, we have:*

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p)$$

$$n = \left(\frac{3}{0.04}\right)^2 (0.01)(0.99) = 56$$

p chart (Fraction Nonconforming)

- *If the in-control value of the fraction nonconforming is **small**, another useful criterion is to choose n **large** enough so that the control chart will have a positive lower control limit.*
- *Since we wish to have*

$$\text{LCL} = p - L\sqrt{\frac{p(1-p)}{n}} > 0$$

- *This implies that*

$$n > \frac{(1-p)}{p} L^2$$

p chart (Fraction Nonconforming)

- *For example, if p = 0.05 and three-sigma limits are used, the sample size must be*

$$n > \frac{(1-p)}{p} L^2$$

$$n > \frac{0.95}{0.05}(3)^2 = 171$$

- *Thus, if $n \geq 172$ units, the control chart will have a positive lower control limit.*

np chart (Number Nonconforming)

- *It is also possible to base a control chart on the **number nonconforming** rather than the fraction nonconforming. This is often called a **number nonconforming (np) control chart**. The parameters of this chart are as follows.*

The *np* Control Chart

$$\begin{aligned} \text{UCL} &= np + 3\sqrt{np(1-p)} \\ \text{Center line } &= np \\ \text{LCL} &= np - 3\sqrt{np(1-p)} \end{aligned} \qquad (7.13)$$

- *If a standard value for p is unavailable, then $\bar{p}$ can be used to estimate p.*

Variable Sample Size

- *In some applications of the control chart for fraction nonconforming, the sample is a 100% inspection of process output over some period of time.*
- *Since different numbers of units could be produced in each period, the control chart would then have a variable sample size.*
- *There are three approaches to constructing and operating a control chart with a variable sample size:*
  - *Variable-Width Control Limits*
  - *Control Limits Based on an Average Sample Size*
  - *The Standardized Control Chart*

Variable-Width Control Limits

- *That is, if the i^{th} sample is of size n_i, then the upper and lower control limits are*

$$\overline{p} \pm 3\sqrt{\overline{p}(1-\overline{p})/n_i}$$

- *Note that the width of the control limits is inversely proportional to the square root of the sample size*

- *For example, consider the data in Table 7.4*

■ **TABLE 7.4**
Purchase Order Data for a Control Chart for Fraction Nonconforming with Variable Sample Size

Sample Number, i	Sample Size, n_i	Number of Nonconforming Units, D_i	Sample Fraction Nonconforming, $\hat{p}_i = D_i/n_i$	Standard Deviation $\hat{\sigma}_{\hat{p}} = \sqrt{\frac{(0.096)(0.904)}{n_i}}$	Control Limits LCL	Control Limits UCL
1	100	12	0.120	0.029	0.009	0.183
2	80	8	0.100	0.033	0	0.195
3	80	6	0.075	0.033	0	0.195
4	100	9	0.090	0.029	0.009	0.183
5	110	10	0.091	0.028	0.012	0.180
6	110	12	0.109	0.028	0.012	0.180
7	100	11	0.110	0.029	0.009	0.183
8	100	16	0.160	0.029	0.009	0.183
9	90	10	0.110	0.031	0.003	0.189
10	90	6	0.067	0.031	0.003	0.189
11	110	20	0.182	0.028	0.012	0.180
12	120	15	0.125	0.027	0.015	0.177
13	120	9	0.075	0.027	0.015	0.177
14	120	8	0.067	0.027	0.015	0.177
15	110	6	0.055	0.028	0.012	0.180
16	80	8	0.100	0.033	0	0.195
17	80	10	0.125	0.033	0	0.195
18	80	7	0.088	0.033	0	0.195
19	90	5	0.056	0.031	0.003	0.189
20	100	8	0.080	0.029	0.009	0.183
21	100	5	0.050	0.029	0.009	0.183
22	100	8	0.080	0.029	0.009	0.183
23	100	10	0.100	0.029	0.009	0.183
24	90	6	0.067	0.031	0.003	0.189
25	90	9	0.100	0.031	0.003	0.189
	2,450	234	2.383			

Variable-Width Control Limits

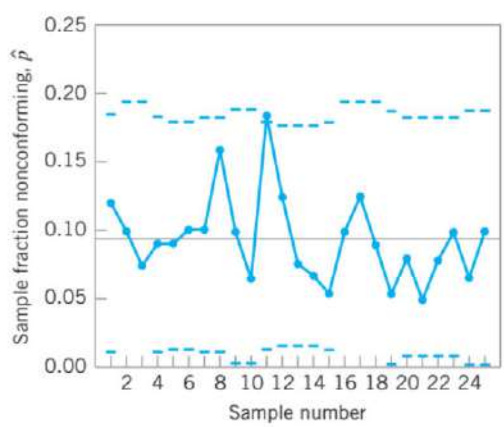
- *Answer. For the 25 samples, we calculate*

$$\overline{p} = \frac{\sum_{i=1}^{25} D_i}{\sum_{i=1}^{25} n_i} = \frac{234}{2{,}450} = 0.096$$

- *Consequently, the center line is at 0.096, and the control limits are*

$$\text{UCL} = \overline{p} + 3\hat{\sigma}_{\hat{p}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$

$$\text{LCL} = \overline{p} - 3\hat{\sigma}_{\hat{p}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$



Control Limits Based on an Average Sample Size

- *The second approach is to base the control chart on an average sample size, resulting in an* ***approximate set of control limits****.*

- *For the purchase order data in Table 7.4, we find that the average sample size is*

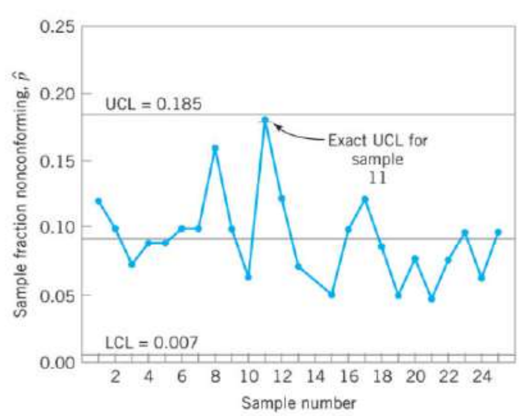
$$\overline{n} = \frac{\sum_{i=1}^{25} n_i}{25} = \frac{2,450}{25} = 98$$

Control Limits Based on an Average Sample Size

- *Therefore, the approximate control limits are*

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.185$$

$$\text{LCL} = \overline{p} - \sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.007$$



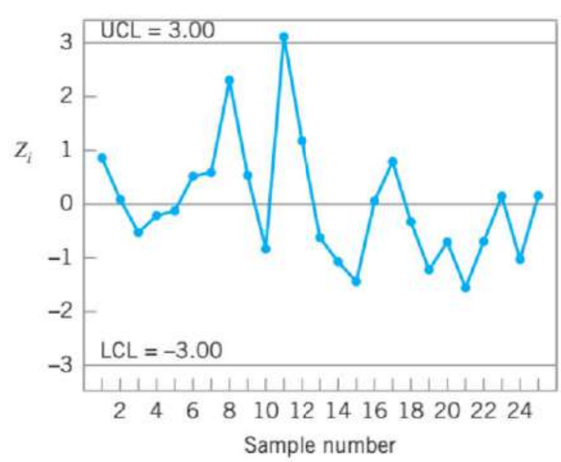
The Standardized Control Chart

- *The third approach to dealing with variable sample size is to use a standardized control chart, where the points are plotted in standard deviation units.*
- *Such a control chart has* ***the center line at zero****, and* ***upper and lower control limits of +3 and −3****, respectively.*
- *The variable plotted on the chart is*

$$Z_i = \frac{\hat{p}_i - p}{\sqrt{\frac{p(1-p)}{n_i}}} \tag{7.14}$$

The Standardized Control Chart

- *The standardized control chart for the purchase order data in Table 7.4 is:*



■ **TABLE 7.5**
Calculations for the Standardized Control Chart in Figure 7.9, $\hat{p} = 0.096$

Sample Number, i	Sample Size, n_i	Number of Nonconforming Units, D_i	Sample Fraction Nonconforming, $\hat{p}_i = D_i/n_i$	Standard Deviation $\hat{\sigma}_p = \sqrt{\frac{(0.096)(0.904)}{n_i}}$	$z_i = \frac{\hat{p}_i - \bar{p}}{\sqrt{\frac{(0.096)(0.904)}{n_i}}}$
1	100	12	0.120	0.029	0.83
2	80	8	0.100	0.033	0.12
3	80	6	0.075	0.033	−0.64
4	100	9	0.090	0.029	−0.21
5	110	10	0.091	0.028	−0.18
6	110	12	0.109	0.028	0.46
7	100	11	0.110	0.029	0.48
8	100	16	0.160	0.029	2.21
9	90	10	0.110	0.031	0.45
10	90	6	0.067	0.031	−0.94
11	110	20	0.182	0.028	3.07
12	120	15	0.125	0.027	1.07
13	120	9	0.075	0.027	−0.78
14	120	8	0.067	0.027	−1.07
15	110	6	0.055	0.028	−1.46
16	80	8	0.100	0.033	0.12
17	80	10	0.125	0.033	0.88
18	80	7	0.088	0.033	−0.24
19	90	5	0.056	0.031	−1.29
20	100	8	0.080	0.029	−0.55
21	100	5	0.050	0.029	−1.59
22	100	8	0.080	0.029	−0.55
23	100	10	0.100	0.029	0.14
24	90	6	0.067	0.031	−0.94
25	90	9	0.100	0.031	0.13

OC Function in p chart

- *The operating-characteristic (or OC) function of the fraction nonconforming control chart is a graphical display of the probability of incorrectly accepting the hypothesis of statistical control (i.e., a type II or b-error) against the process fraction nonconforming*

$$\begin{aligned}\beta &= P\{\hat{p} < \mathrm{UCL}|p\} - P\{\hat{p} \le \mathrm{LCL}|p\} \\ &= P\{D < n\mathrm{UCL}|p\} - P\{D \le n\mathrm{LCL}|p\}\end{aligned} \quad (7.15)$$

Average Run Lengths (ARLs) in p chart

$$\text{ARL} = \frac{1}{P(\text{sample point plots out of control})}$$

$$\text{ARL}_0 = \frac{1}{\alpha}$$

$$\text{ARL}_1 = \frac{1}{1-\beta}$$

Control Charts for Nonconformities (Defects)

- *A nonconforming item is a unit of product that does not satisfy one or more of the specifications for that product*
- *Consequently, a nonconforming item will contain at least one nonconformity*
- *Depending on their nature and severity, it is quite possible for a unit to contain several nonconformities and **not** be classified as nonconforming*

Control Charts for Nonconformities (Defects)

- *So, nonconformities are acceptable to a level, not more!*
- *It is possible to develop control charts for either the **total number of nonconformities** in a unit or **the average number of nonconformities** per unit.*
- *These control charts usually assume that the occurrence of nonconformities in samples of constant size is well modeled by the **Poisson distribution**.*

Control Charts for Nonconformities (Defects)

- *Consider the occurrence of nonconformities in an inspection unit of product occurs according to the Poisson distribution:*

$$p(x) = \frac{e^{-c} c^x}{x!} \qquad x = 0,\ 1,\ 2\ ,\ \ldots$$

- *We recall that both the mean and variance of the Poisson distribution are the parameter c.*

c chart (Nonconformities)

- *Therefore, a* ***control chart for nonconformities*** *(or defects), or* ***c chart*** *with three-sigma limits would be defined as follows*

Control Chart for Nonconformities: Standard Given

$$\begin{aligned} \text{UCL} &= c + 3\sqrt{c} \\ \text{Center line } &= c \\ \text{LCL} &= c - 3\sqrt{c} \end{aligned} \qquad (7.16)$$

c chart (Nonconformities)

- *If no standard is given, then c may be estimated as the observed average number of nonconformities in a preliminary sample of inspection units*

Control Chart for Nonconformities: No Standard Given

$$\begin{aligned} \text{UCL} &= \bar{c} + 3\sqrt{\bar{c}} \\ \text{Center line } &= \bar{c} \\ \text{LCL} &= \bar{c} - 3\sqrt{\bar{c}} \end{aligned} \qquad (7.17)$$

c chart (Nonconformities)

- *Example 7.3 Table 7.7 presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Set up a c chart for these data.*

TABLE 7.7
Data on the Number of Nonconformities in Samples of 100 Printed Circuit Boards

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

c chart (Nonconformities)

- *Answer. Since the 26 samples contain 516 total nonconformities, we estimate c by*

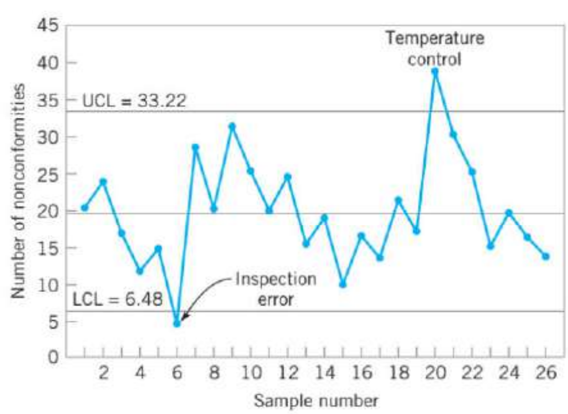
$$\bar{c} = \frac{516}{26} = 19.85$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line } = \bar{c} = 19.85$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$

Two points plot outside the control limits: samples 6 and 20.



u chart (Nonconformities per unit)

- *We would often prefer to use several inspection units in the sample, thereby increasing the area of opportunity for the occurrence of nonconformities.*
- *Sample size should be selected according to the statistical considerations and many vary each time*
- *If we find x total nonconformities in a sample of n inspection units, then the average number of nonconformities per inspection unit is*

$$u = \frac{x}{n}$$

- *Note that x is a Poisson random variable*

u chart (Nonconformities per unit)

- *The parameters of the control chart for the average number of nonconformities per unit are as follows*

Control Chart for Average Number of Nonconformities per Unit

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \\ \text{Center line } &= \bar{u} \\ \text{LCL} &= \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \end{aligned} \qquad (7.19)$$

u chart (Nonconformities per unit)

- *Example 7.4 Draw a u chart for the following data:*

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.0740$$

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 + 3\sqrt{\frac{0.0740}{50}} = 0.1894$$

$$\text{Center line } = \bar{u} = 1.93$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 - 3\sqrt{\frac{0.0740}{50}} = -0.0414$$

Data on Number of Shipping Errors in a Supply Chain Network

Sample Number (week), i	Sample Size, n	Total Number of Errors (Nonconformities), x_i	Average Number of Errors (Nonconformities) per Unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48

u chart (Nonconformities per unit)

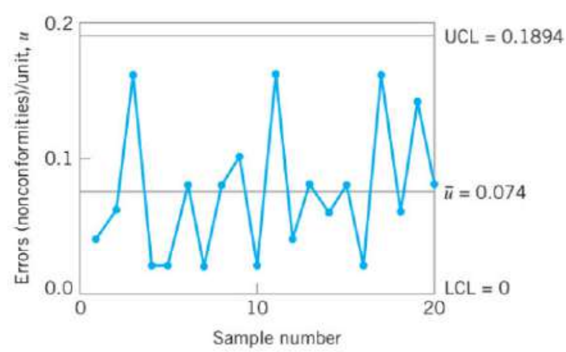
- *Example 7.4 Draw a u chart for the following data:*

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.0740$$

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 + 3\sqrt{\frac{0.0740}{50}} = 0.1894$$

$$\text{Center line } = \bar{u} = 1.93$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 - 3\sqrt{\frac{0.0740}{50}} = -0.0414$$



Procedures with Variable Sample Size

- *Use control limits based on an average sample size*

$$\overline{n} = \sum_{i=1}^{m} n_i / m$$

- *Use a standardized control chart (this is the preferred option)*
  LCL = −3 and UCL = +3 and the center line at zero.

$$Z_i = \frac{u_i - \overline{u}}{\sqrt{\frac{\overline{u}}{n_i}}}$$

Alternative Probability Models for Count Data

- *Most applications of the **c chart** assume that the **Poisson distribution** is the correct probability model underlying the process.*

- *It is not the only distribution that could be utilized as a model of **count** or nonconformities per unit-type data*

- *Various types of phenomena can produce distributions of defects that are not well modeled by the Poisson distribution.*
  - *In the Poisson distribution, the mean and the variance are equal. When the sample data indicate that **the sample variance is substantially different from the mean**, the Poisson assumption is likely to be **inappropriate**.*

Alternative Probability Models for Count Data

- *Kaminsky et al. (1992) have proposed control charts for counts based on the* ***geometric distribution****.*
- *The probability model that they use for the geometric distribution is*

$$p(x) = p(1-p)^{x-a} \text{ for } x = a, a+1, a+2, \ldots$$

where a is the known minimum possible number of events.

- *The two statistics that can be used to form a control chart are*
  - *the* ***total*** *number of events* $T = x_1 + x_2 + \ldots + x_n$
  - *the* ***average*** *number of events* $\bar{x} = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$

Alternative Probability Models for Count Data

- *Suppose that the data from the process are available as a subgroup of size n, say* x_1, x_2, $\ldots x_n$
- *These observations are* ***independently and identically distributed*** *observations from a* ***geometric distribution*** *when the process is stable (in control).*
- *We know that the sum of independently and identically distributed geometric random variables is a* ***negative binomial*** *random variable.*

Alternative Probability Models for Count Data

- *The mean and variance of the* ***total*** *number of events T are*

$$\mu_T = n\left(\frac{1-p}{p} + a\right) \qquad \sigma_T^2 = \frac{n(1-p)}{p^2}$$

- *The mean and variance of the* ***average*** *number of events are*

$$\mu_{\bar{x}} = \frac{1-p}{p} + a \qquad \sigma_{\bar{x}}^2 = \frac{1-p}{np^2}$$

Alternative Probability Models for Count Data

- *Kaminsky et al. (1992) refer to ...*
- *The control chart for the* ***total*** *number of events as a* **g chart**
- *The control chart for the* ***average*** *number of events as an* **h chart**

- *The center lines and control limits for each chart are shown in the following display.*

g and h chart

- *The center lines and control limits for each chart are shown in the following display.*

g and h Control Charts, Standards Given

	Total number of events chart, g chart	Average number of events chart, h chart
Upper control limit (UCL)	$n\left(\frac{1-p}{p}+a\right)+L\sqrt{\frac{n(1-p)}{p^2}}$	$\frac{1-p}{p}+a+L\sqrt{\frac{1-p}{np^2}}$
Center line (CL)	$n\left(\frac{1-p}{p}+a\right)$	$\frac{1-p}{p}+a$
Lower control limit (LCL)	$n\left(\frac{1-p}{p}+a\right)-L\sqrt{\frac{n(1-p)}{p^2}}$	$\frac{1-p}{p}+a-L\sqrt{\frac{1-p}{np^2}}$

g and h chart

- *While we have assumed that* ***a is known****, in most situations the parameter* ***p will likely be unknown****.*

g and h Control Charts, No Standards Given

	Total number of events chart, g chart	Average number of events chart, h chart
Upper control limit (UCL)	$\bar{t}+L\sqrt{n\left(\frac{\bar{t}}{n}-a\right)\left(\frac{\bar{t}}{n}-a+1\right)}$	$\frac{\bar{t}}{n}+\frac{L}{\sqrt{n}}\sqrt{\left(\frac{\bar{t}}{n}-a\right)\left(\frac{\bar{t}}{n}-a+1\right)}$
Center line (CL)	$\bar{t}$	$\frac{\bar{t}}{n}$
Lower control limit (LCL)	$\bar{t}-L\sqrt{n\left(\frac{\bar{t}}{n}-a\right)\left(\frac{\bar{t}}{n}-a+1\right)}$	$\frac{\bar{t}}{n}-\frac{L}{\sqrt{n}}\sqrt{\left(\frac{\bar{t}}{n}-a\right)\left(\frac{\bar{t}}{n}-a+1\right)}$

Demerit Systems

- *With complex products such as automobiles, computers, or major appliances, we usually find that many* ***different types of nonconformities*** *or defects can occur.*
- ***Not all*** *of these types of defects are* ***equally important***
- *A unit of product having one very serious defect would probably be classified as nonconforming to requirements, but a unit having several minor defects might not necessarily be nonconforming*
- *For example, we can classify nonconformities as either* ***functional defects*** *or* ***appearance defects*** *if a two-class system is preferred.*
- ***Demerit systems for attribute data*** *can be of value in these situations.*

Demerit Systems

- *One possible demerit scheme is defined as follows*

Class A Defects—Very Serious. The unit is either completely unfit for service, or will fail in service in such a manner that cannot be easily corrected in the field, or will cause personal injury or property damage.

Class B Defects—Serious. The unit will possibly suffer a Class A operating failure, or will certainly cause somewhat less serious operating problems, or will certainly have reduced life or increased maintenance cost.

Class C Defects—Moderately Serious. The unit will possibly fail in service, or cause trouble that is less serious than operating failure, or possibly have reduced life or increased maintenance costs, or have a major defect in finish, appearance, or quality of work.

Class D Defects—Minor. The unit will not fail in service but has minor defects in finish, appearance, or quality of work.

Demerit Systems

- *Let c_{iA}, c_{iB}, c_{iC}, and c_{iD} represent the number of Class A, Class B, Class C, and Class D defects, respectively, in the i^{th} inspection unit*
- *We assume that **each class of defect is independent**, and the occurrence of defects in each class is well modeled by a **Poisson distribution***
- *Then we define the number of **demerits** in the inspection unit as*

$$d_i = 100c_{i\mathrm{A}} + 50c_{i\mathrm{B}} + 10c_{i\mathrm{C}} + c_{i\mathrm{D}}$$

- *The demerit weights of Class A—100, Class B—50, Class C—10, and Class D—1 are used fairly widely in practice.*
- *However, any reasonable set of weights appropriate for a specific problem may also be used.*

Demerit Systems

- *Suppose that a sample of n inspection units is used.*
- *Then the number of demerits per unit is*

$$u_i = \frac{D}{n}$$

$$D = \sum_{i=1}^{n} d_i$$

$$d_i = 100c_{i\mathrm{A}} + 50c_{i\mathrm{B}} + 10c_{i\mathrm{C}} + c_{i\mathrm{D}}$$

Demerit Systems

- *Since u_i is a linear combination of independent Poisson random variables, the statistics u_i could be plotted on a control chart with the following parameters:*

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\hat{\sigma}_u \\ \text{Center line } &= \bar{u} \\ \text{LCL} &= \bar{u} - 3\hat{\sigma}_u \end{aligned} \tag{7.23}$$

where

$$\bar{u} = 100\bar{u}_A + 50\bar{u}_B + 10\bar{u}_C + \bar{u}_D \tag{7.24}$$

and

$$\hat{\sigma}_u = \left[\frac{(100)^2\bar{u}_A + (50)^2\bar{u}_B + (10)^2\bar{u}_C + \bar{u}_D}{n}\right]^{1/2} \tag{7.25}$$

Control Charts for Attributes

	p (fraction)	**np (number of nonconforming)**	**c (count of nonconformances)**	**u (count of nonconformances/unit)**
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p}+3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}+3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}+3\sqrt{\bar{c}}$	$\bar{u}+3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p}-3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}-3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}-3\sqrt{\bar{c}}$	$\bar{u}-3\sqrt{\frac{\bar{u}}{n}}$
Notes	If n varies, use $\bar{n}$ or individual n_i	n must be a constant	n must be a constant	If n varies, use $\bar{n}$ or individual n_i

Control Charts for Attributes

Standardized Attributes Control Charts Suitable for Short Production Runs

Attribute	Target Value	Standard Deviation	Statistic to Plot on the Control Chart
$\hat{p}_i$	$\bar{p}$	$\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$Z_i = \frac{\hat{p}_i - \bar{p}}{\sqrt{\bar{p}(1-\bar{p})/n}}$
$n\hat{p}_i$	$n\bar{p}$	$\sqrt{n\bar{p}(1-\bar{p})}$	$Z_i = \frac{n\hat{p}_i - n\bar{p}}{\sqrt{n\bar{p}(1-\bar{p})}}$
c_i	$\bar{c}$	$\sqrt{\bar{c}}$	$Z_i = \frac{c_i - \bar{c}}{\sqrt{\bar{c}}}$
u_i	$\bar{u}$	$\sqrt{\bar{u}/n}$	$Z_i = \frac{u_i - \bar{u}}{\sqrt{\bar{u}/n}}$

The Quality Chronicle

	Attribute	***Variable***
Input / Output	*Attribute Sampling Plans*	*Variable Sampling Plans*
Process	*Attribute Control Charts*	*Variable Control Charts*