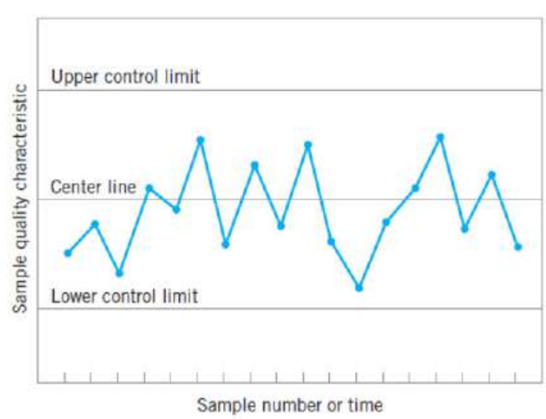
Chapter 5

Statistical Basis of the Control Chart

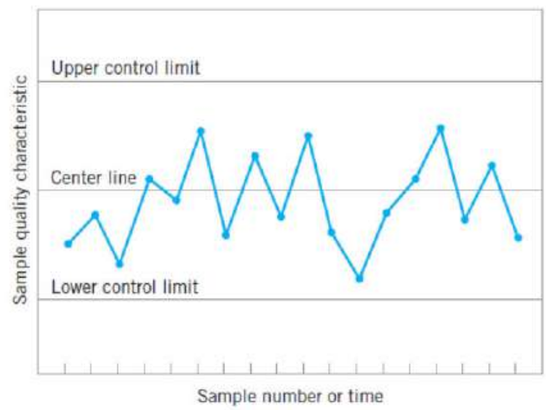
Statistical Basis of the Control Chart

- *The **control chart** is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time*
- *The chart contains a **center line** that represents the average value of the quality characteristic corresponding to the in-control state*



Statistical Basis of the Control Chart

- *The **Upper Control Limit (UCL)** and **Lower Control Limit (LCL)**, are control limits to show if the process is in control, nearly all of the sample points will fall between them, and no action is necessary*
- *A point that plots outside of the control limits is interpreted as evidence that the process is **out of control**, and investigation and corrective action are required to find and eliminate the assignable cause or causes responsible for this behavior*

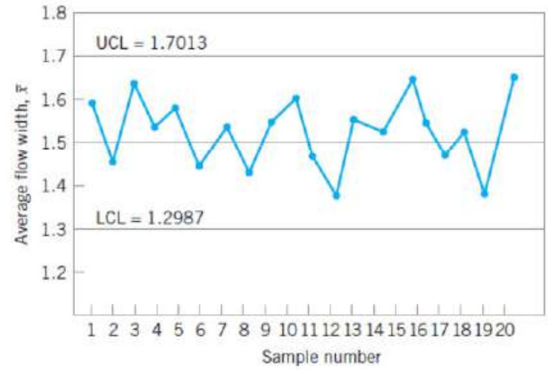


Statistical Basis of the Control Chart

- *Consider the manufacturing process of a silicon wafer, an important quality characteristic in hard bake is the **width** of the resist.*
- *Suppose that width can be controlled at a mean of 1.5 microns, and it is known that the standard deviation of flow width is 0.15 microns.*
- *Every hour, a sample of **five** wafers is taken*
- *According to the gathered data, the related control chart is presented in the following*

Statistical Basis of the Control Chart

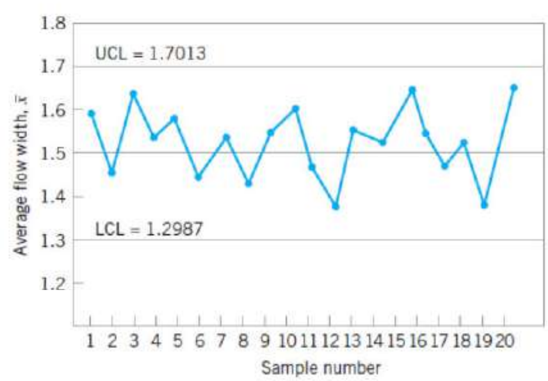
- *Because this control chart utilizes the sample average to monitor $\bar{x}$ the process mean, it is usually called an* ***$\overline{x}$ control chart***
- *Note that all of the plotted points fall inside the control limits, so the chart indicates that the process is considered to be* ***in statistical control****.*



Statistical Basis of the Control Chart

- *Now lets see how the control limits were determined*
- *The process mean is μ=1.5 microns, and the process standard deviation is σ=0.15 microns.*
- *Now as samples of size n=5 are taken, the standard deviation of the sample average is*

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.15}{\sqrt{5}} = 0.0671$$



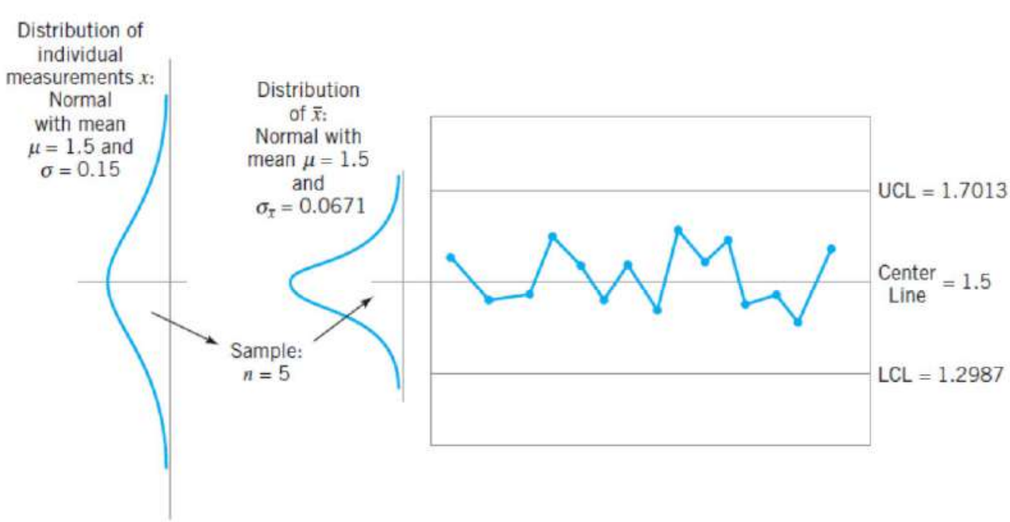
Statistical Basis of the Control Chart

- *Therefore, if the process is in control with a mean flow width of 1.5 microns, then by using the central limit theorem to assume that is approximately normally distributed*
- *We would expect* ***100(1-α)%*** *of the sample means* $\bar{x}$ *to fall between* $\mathbf{1.5+Z_{\alpha/2}(0.0671)}$ *and* $\mathbf{1.5- Z_{\alpha/2}(0.0671)}$
- *In this step, we arbitrarily choose the constant* $Z_{\alpha/2}$ *to be 3, so that the upper and lower control limits become (These are typically called* ***three-sigma control limits****)*

$$\mathrm{UCL} = 1.5 + 3(0.0671) = 1.7013$$

$$\mathrm{LCL} = 1.5 - 3(0.0671) = 1.2987$$

Statistical Basis of the Control Chart



Statistical Basis of the Control Chart

- *Now, we give a general model for control charts*
- *Let w be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of w is μ_w and the standard deviation of w is σ_w.*
- *Then the center line, the upper control limit, and the lower control limit become:*

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line} &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned} \qquad (5.1)$$

Statistical Basis of the Control Chart

- *L is the "distance" of the control limits from the center line*
- *This general theory of control charts was first proposed by **Walter A. Shewhart**, and control charts developed according to these principles are often called **Shewhart control charts**.*

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line} &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned} \qquad (5.1)$$

Choice of Control Limits

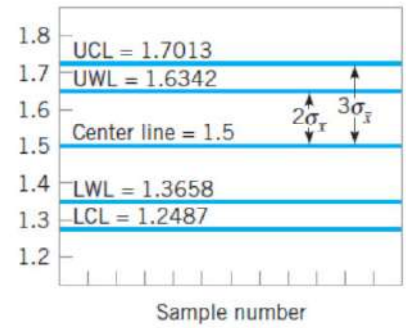
- *Specifying the control limits is one of the critical decisions that must be made in designing a control chart*
- *By moving the control limits* ***farther from the center line****, we* ***decrease the risk of a type I error****—that is, the risk of a point falling beyond the control limits, indicating an out-of-control condition when no assignable cause is present. However, widening the control limits will also increase the risk of a type II error—that is, the risk of a point falling between the control limits when the process is really out of control.*
- *If we move the control limits* ***closer to the center line****, the opposite effect is obtained:* ***The risk of type I error is increased****, while* ***the risk of type II error is decreased****.*

Choice of Control Limits

Table of error types		*Null hypothesis (H_0) is*	
		True	*False*
Decision about null hypothesis (H_0)	*Fail to reject*	*Correct inference (true negative) (probability = $1-\alpha$)*	*Type II error (false negative) (probability = β)*
	Reject	*Type I error (false positive) (probability = α)*	*Correct inference (true positive) (probability = $1-\beta$)*

Choice of Control Limits

- *Three-sigma are the usual* ***action limits***
- *Two-sigma are called* ***warning limits***



Sample Size

- *In designing a control chart, we must specify both the* ***sample size*** *and the* ***frequency of sampling***
- *In general, larger samples will make it easier to detect small shifts in the process*
- *When choosing the sample size, we must keep in mind the size of the shift that we are trying to detect.*
- *If the process shift is relatively large, then we use smaller sample sizes than those that would be employed if the shift of interest were relatively small*

Sample Size

- *We must also determine the frequency of sampling.*
- *The most desirable situation from the point of* ***view of detecting shifts*** *would be to take* ***large samples very frequently***
- *This is usually* ***not*** *economically feasible*
- *The general problem is one of allocating sampling effort. That is, either we take small samples at short intervals or larger samples at longer intervals.*
- *Current industry practice tends to favor* ***smaller, more frequent samples****, particularly in high-volume manufacturing processes, or where a great many types of assignable causes can occur*

Sampling Frequency

- *A way to evaluate the decisions regarding sample size and sampling frequency is the Average Run Length (ARL)*
- *The ARL is the average number of points that must be plotted before a point indicates an out-of-control condition*
- *If the process observations are* ***uncorrelated****, then for any Shewhart control chart, the ARL can be calculated easily from*

$$\mathrm{ARL} = \frac{1}{p} \tag{5.2}$$

where p is the probability that any point exceeds the control limits

Sampling Frequency

- *To illustrate, for the $\bar{x}$ chart with three-sigma limits, $p = 0.0027$ is the probability that a single point falls outside the limits when the process is in control.*
- *Therefore, the average run length of the chart when the process is in control (called ARL_0) is*

$$\mathrm{ARL}_0 = \frac{1}{p} = \frac{1}{0.0027} = 370$$

- *That is, even if the process remains in control, an out-of-control signal will be generated every 370 samples, on the average.*

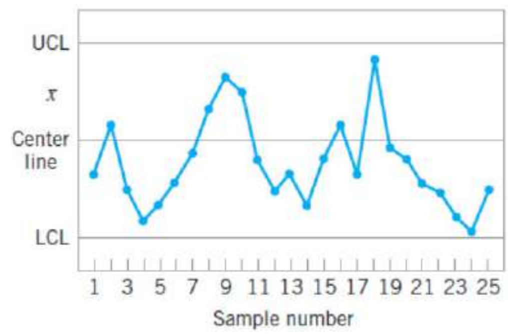
Sampling Frequency

- *It is also occasionally convenient to express the performance of the control chart in terms of its Average Time to Signal (ATS)*
- *If samples are taken at fixed intervals of time that are h hours apart, then*

$$\mathrm{ATS} = \mathrm{ARL}h \qquad (5.3)$$

Analysis of Patterns on Control Charts

- *Patterns on control charts must be assessed.*
- *A control chart may indicate an out-of-control condition when one or more points fall beyond the control limits or when the plotted points exhibit some nonrandom pattern of behavior*
  - *For example, consider the $\bar{x}$ chart shown in the figure. Although all 25 points fall within the control limits, the points do not indicate statistical control because their pattern is very nonrandom in appearance.*



Analysis of Patterns on Control Charts

- *The problem of **pattern recognition** is recognizing systematic or nonrandom patterns on the control chart and identifying the reason for this behavior*
- *The ability to interpret a particular pattern in terms of assignable causes requires **experience** and **knowledge** of the process*
- *The Western Electric Statistical Quality Control Handbook (1956) suggests a set of decision rules for detecting nonrandom patterns on control charts*

Analysis of Patterns on Control Charts

Some Sensitizing Rules for Shewhart Control Charts

Standard Action Signal:	**1.** One or more points outside of the control limits	Western Electric Rules
	2. Two of three consecutive points outside the two-sigma warning limits but still inside the control limits	
	3. Four of five consecutive points beyond the one-sigma limits	
	4. A run of eight consecutive points on one side of the center line	
	5. Six points in a row steadily increasing or decreasing	
	6. Fifteen points in a row in zone C (both above and below the center line)	
	7. Fourteen points in a row alternating up and down	
	8. Eight points in a row on both sides of the center line with none in zone C	
	9. An unusual or nonrandom pattern in the data	
	10. One or more points near a warning or control limit	

Analysis of Patterns on Control Charts

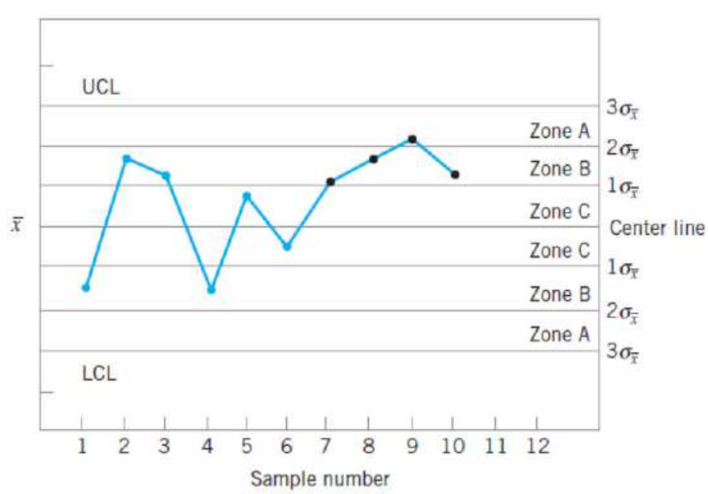


FIGURE 5.15 The Western Electric or zone rules, with the last four points showing a violation of rule 3.

Interpretation of $\bar{x}$ and R Charts

- *We review some of the process characteristics that may produce the patterns*
- *Additional information on the interpretation of patterns on control charts is in the Western Electric Statistical Quality Control Handbook (1956, pp. 149–183)*

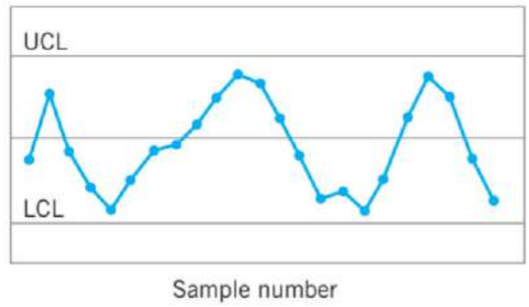


FIGURE 6.8 Cycles on a control chart.

Interpretation of $\bar{x}$ and R Charts

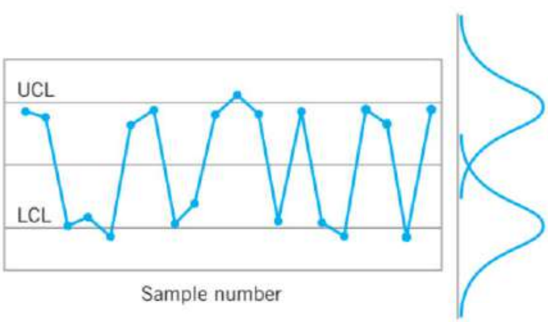


FIGURE 6.9 A mixture pattern.

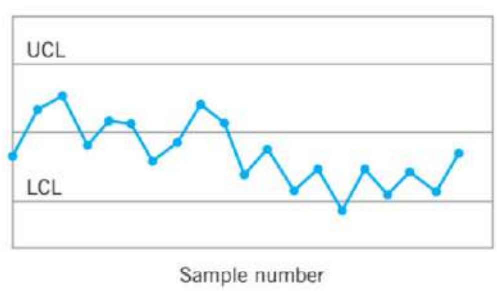


FIGURE 6.10 A shift in process level.

Interpretation of $\bar{x}$ and R Charts

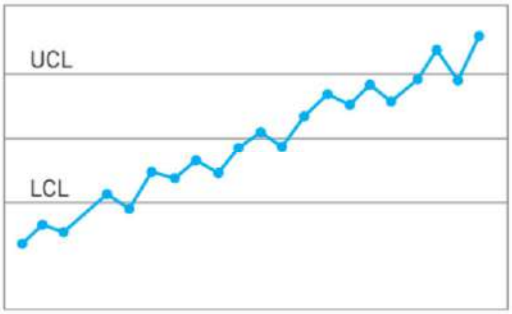


FIGURE 6.11 A trend in process level.

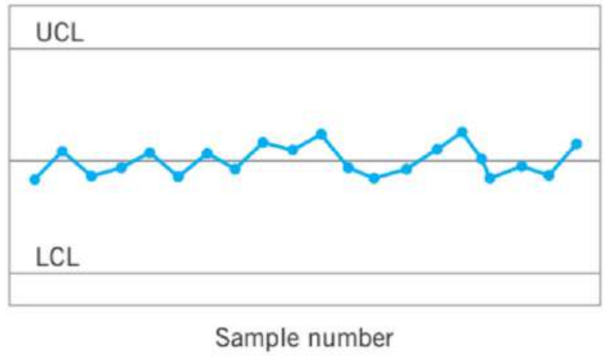


FIGURE 6.12 A stratification pattern.

Phase I and Phase II of Control Chart Application

- ***Phase I***
- *A set of process data is gathered and analyzed all at once in a retrospective analysis, constructing trial control limits to determine if the process has been in control over the period of time during which the data were collected, and to see if reliable control limits can be established to monitor future production.*
- *This is typically the first thing that is done when control charts are applied to any process.*
- *Control charts in phase I primarily assist operating personnel in bringing the process into a state of statistical control.*

Phase I and Phase II of Control Chart Application

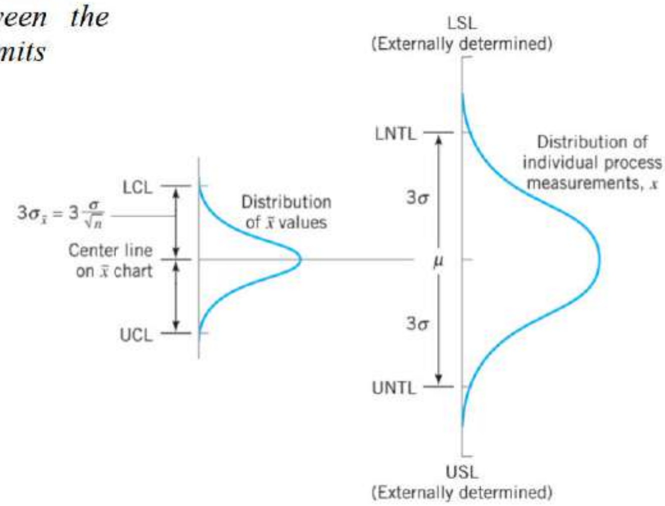
- ***Phase II***
- *Begins after we have a "clean" set of process data gathered under stable conditions and representative of in-control process performance*
- *In phase II, we use the control chart to monitor the process by comparing the sample statistic for each successive sample as it is drawn from the process to the control limits*

Estimating Process Capability

- ***Control limits*** *describe what a process is capable of producing (sometimes referred to as the "voice of the process")*
- ***Tolerances limits*** *represent the natural variability of the process (usually set at 3-sigma from the mean)*
- ***Specification limits*** *describe how the product should perform to meet the customer's expectations (referred to as the "voice of the customer")*

Estimating Process Capability

- *Remember that there is **no** mathematical or statistical relationship between the control limits and specification limits*



Estimating Process Capability

No.	Control limits	Specifications limits
1	Voice of process	Voice of customer
2	Calculated from the process data	Defined by the customer
3	Focus on location, spread, and width	Focus on meeting the requirements
4	Show the variation in the performance of the process	Represent the desired performance of the process
5	Displayed on control charts	Displayed on histogram
6	Applied to subgroup	Applied to items
7	Helps to reduce internal rejection	Helps to reduce customer rejection
8	Guide for process actions	Separate good items from bad
9	What the process is doing	What we want the process to do

Estimating Process Capability

- We define the Process Capability Ratio (PCR) C_p as below:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \qquad (6.11)$$

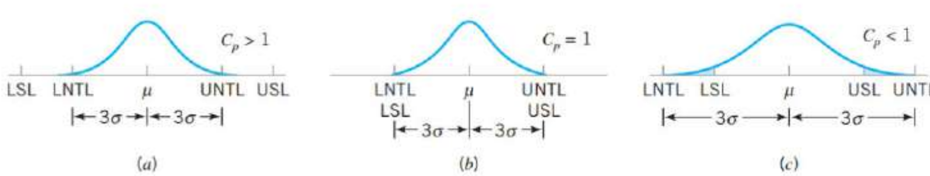
the ability of an engineering process to produce an output within specification limits

- Since σ is usually **unknown**, we must replace it with an estimate.
- We use $\hat{\sigma} = \bar{R}/d_2$ as an estimate of σ.

Estimating Process Capability

- For the hard-bake process:
- The specification limits on flow width are 1.50 ± 0.50 microns.
- The control chart data may be used to describe the capability of the process to produce wafers relative to these specifications.

$$\hat{C}_p = \frac{2.00 - 1.00}{6(0.1398)} = \frac{1.00}{0.8388} = 1.192$$



Estimating Process Capability

- *For one-sided specifications, one-sided process-capability ratios are used.*
- *One-sided PCRs are defined as follows:*

$$C_{pu} = \frac{\text{USL} - \mu}{3\sigma} \qquad \text{(upper specification only)} \qquad (8.7)$$

$$C_{pl} = \frac{\mu - \text{LSL}}{3\sigma} \qquad \text{(lower specification only)} \qquad (8.8)$$