Chapter 4

Variables Sampling Plans

Variables Sampling Plans

- *When actual quantitative information can be measured on sampled items, rather than simply classifying them as conforming or nonconforming.*
- *When a lot is rejected, the measurements in relation to the specification limits give additional information to the supplier and may help to prevent rejected lots in the future.*
- *The disadvantage of variables sampling plans is that they are based on the assumption that the measurements are normally distributed*
- *There are three methods of*
  - *k-method*
  - *M-method*
  - *Gauge R&R*

k-Method – LSL Known SD

- *To define a variables sampling plan the number of samples n and an acceptance constant k must be determined.*
- *A lot would be accepted if*

$$(\bar{x} - LSL)/\sigma > k$$

- *where $\bar{x}$ is the sample mean and σ is the standard deviation of the measurements*
- *The mean and standard deviation are assumed to be known from past experience.*

k-Method– LSL Known SD

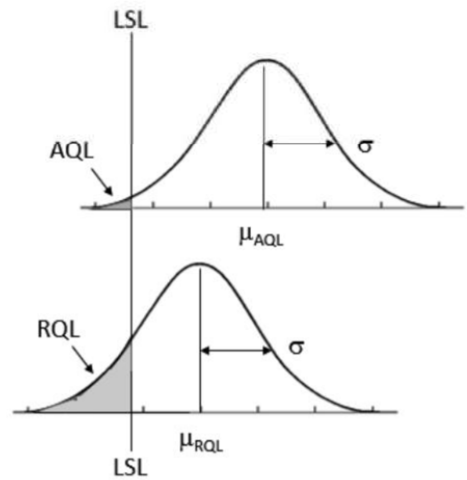
- *In terms of the statistical theory of hypothesis testing, accepting the lot would be equivalent to failing to reject the null hypothesis:*

$$H_0 : \mu \geq \mu_{AQL}$$

$$H_a : \mu < \mu_{AQL}$$

k-Method– LSL Known SD

- *When the measurements are assumed to be normally distributed with a lower specification limit LSL, the AQL and the RQL in terms of proportion of items below the LSL can be visualized as areas under the normal curve to the left of the LSL as shown*



k-Method– LSL Known SD

- *If the producer's risk is α and the consumer's risk is β, then*

$$P\left(\frac{\bar{x} - LSL}{\sigma} > k \mid \mu = \mu_{AQL}\right) = 1 - \alpha \qquad (3.1)$$

$$P\left(\frac{\bar{x} - LSL}{\sigma} > k \mid \mu = \mu_{RQL}\right) = \beta \qquad (3.2)$$

k-Method– LSL Known SD

- By multiplying both sides of the inequality inside the parenthesis by $\sqrt{n}$ subtracting $\frac{\mu_{AQL}}{\sigma/\sqrt{n}}$ from each side, and adding $\frac{LSL}{\sigma/\sqrt{n}}$ to each side, it can be seen that

$$P\left(\frac{\bar{x} - LSL}{\sigma} > k \mid \mu = \mu_{AQL}\right) = 1 - \alpha$$

$$\frac{\bar{x} - LSL}{\sigma} > k \Rightarrow \frac{\bar{x} - \mu_{AQL}}{\sigma/\sqrt{n}} > k\sqrt{n} + \frac{LSL - \mu_{AQL}}{\sigma/\sqrt{n}}$$

k-Method – LSL Known SD

- So

$$P\left(Z > k\sqrt{n} + \frac{LSL - \mu_{AQL}}{\sigma/\sqrt{n}}\right) = 1 - \alpha,$$

$$P\left(Z < k\sqrt{n} + \frac{LSL - \mu_{AQL}}{\sigma/\sqrt{n}}\right) = \alpha \tag{3.4}$$

$$k\sqrt{n} + \frac{LSL - \mu_{AQL}}{\sigma/\sqrt{n}} = Z_\alpha$$

$$k = \frac{Z_\alpha}{\sqrt{n}} - \frac{LSL - \mu_{AQL}}{\sigma} = \frac{Z_\alpha}{\sqrt{n}} - Z_{AQL} \tag{3.5}$$

k-Method – LSL Known SD

- *Performing the same manipulations with the inequality in second equation it can be shown that*

$$k = \frac{Z_{1-\beta}}{\sqrt{n}} - Z_{RQL} \qquad (3.6)$$

- *Equating the solution for k in the next to last line in Equation 3.5 with the solution for k in Equation 3.6 and solving for n, it can be seen that*

$$\frac{Z_\alpha}{\sqrt{n}} - Z_{AQL} = \frac{Z_{1-\beta}}{\sqrt{n}} - Z_{RQL}$$

$$n = \left(\frac{Z_\alpha - Z_{1-\beta}}{Z_{AQL} - Z_{RQL}}\right)^2 \qquad (3.7)$$

k-Method – LSL Known SD

- *So, conducting the sampling plan on a lot consists of the following steps:*

1. *Take a random sample of n items from the lot*
2. *Measure the critical characteristic x on each sampled item*
3. *Calculate the mean measurement* $\bar{x}$
4. *Compare* $\frac{\bar{x}-LSL}{\sigma}$ *to the acceptance constant k*
5. *If* $\frac{\bar{x}-LSL}{\sigma} > k$*, accept the lot, otherwise reject the lot*

k-Method – LSL Unknown SD

- *When the standard deviation is unknown, conducting the sampling plan on a lot of material consists of the following steps:*

1. *Take a random sample of n items from the lot*
2. *Measure the critical characteristic x on each sampled item*
3. *Calculate the mean measurement $\bar{x}$, and the sample standard deviation s*
4. *Compare $\frac{\bar{x}-LSL}{s}$ to the acceptance constant k*
5. *If $\frac{\bar{x}-LSL}{s} > k$, accept the lot, otherwise reject the lot*

k-Method – USL Known SD

- *When the standard deviation is known, conducting the sampling plan on a lot of material consists of the following steps:*

1. *Take a random sample of n items from the lot*
2. *Measure the critical characteristic x on each sampled item*
3. *Calculate the mean measurement $\bar{x}$*
4. *Compare $\frac{USL-\bar{x}}{\sigma}$ to the acceptance constant k*
5. *If $\frac{USL-\bar{x}}{\sigma} > k$, accept the lot, otherwise reject the lot*

k-Method – USL Unknown SD

- *When the standard deviation is unknown, conducting the sampling plan on a lot of material consists of the following steps:*

1. *Take a random sample of n items from the lot*
2. *Measure the critical characteristic x on each sampled item*
3. *Calculate the mean measurement $\bar{x}$, and the sample standard deviation s*
4. *Compare $\frac{USL-\bar{x}}{s}$ to the acceptance constant k*
5. *If $\frac{\bar{x}-LSL}{s} > k$, accept the lot, otherwise reject the lot*

MIL STD 414

- *The standard was introduced in 1957.*
- *MIL STD 414 is a lot-by-lot acceptance-sampling plan for variables.*
- *The focal point of this standard is the acceptable quality level (AQL), which ranges from 0.04% to 15%.*
- *The standard was also adopted by the International Organization for Standardization as ISO 3951.*

MIL STD 414

- *MIL STD 414 is divided into four sections:*
  - ***Section A** is a general description of the sampling plans, including definitions, sample-size code letters, and OC curves.*
  - ***Section B** gives variables-sampling plans based on the sample standard deviation for the case in which the process or lot variability is unknown.*
  - ***Section C** presents variables sampling plans based on the sample range method.*
  - ***Section D** gives variables-sampling plans for the case where the process standard deviation is known.*

Use of the Tables

- *Example. Consider there are soft-drink bottler with the **lower specification limit** on bursting strength is **225 psi**.*
- *We use **inspection level IV**.*
- *Suppose that the **AQL** at this specification limit is **1%**.*
- *Let us suppose that bottles are shipped in **lots of size 100,000**.*
- *Find a variables sampling plan that uses MIL STD 414.*
- *Assume that the lot standard deviation is **unknown**.*

Use of the Tables

- *Answer.*
- *From Table 16.1, if we use inspection level IV, the sample size code letter is O.*
- *From Table 16.2 we find that sample size code letter O implies a sample size of n = 100.*
- *For an acceptable quality level of 1%, on normal inspection, the value of k is 2.00.*
- *If tightened inspection is employed, the appropriate value of k is 2.14.*
- *The AQL values for normal inspection are indexed at the top of the table, and the AQL values for tightened inspection are indexed from the bottom of the table.*

TABLE 16.1

Sample-Size Code Letters (MIL STD 414, Table A.2)

	Inspection Levels				
Lot Size	**I**	**II**	**III**	**IV**	**V**
3 to 8	B	B	B	B	C
9 to 15	B	B	B	B	D
16 to 25	B	B	B	C	E
26 to 40	B	B	B	D	F
41 to 65	B	B	C	E	G
66 to 110	B	B	D	F	H
111 to 180	B	C	E	G	I
181 to 300	B	D	F	H	J
301 to 500	C	E	G	I	K
501 to 800	D	F	H	J	L
801 to 1,300	E	G	I	K	L
1,301 to 3,200	F	H	J	L	M
3,201 to 8,000	G	I	L	M	N
8,001 to 22,000	H	J	M	N	O
22,001 to 110,000	I	K	N	O	P
110,001 to 550,000	I	K	O	P	Q
550,001 and over	I	K	P	Q	Q

TABLE 16.2

Master Table for Normal and Tightened Inspection for Plans Based on Variability Unknown (Standard Deviation Method) (Single-Specification Limit—Form 1)(MIL STD 414, Table B.1)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (normal inspection)													
		.04	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	3	↓	↓	↓	↓	↓	↓	↓	▼	▼	1.12	.958	.765	.566	.341
C	4	↓	↓	↓	↓	↓	↓	↓	1.45	1.34	1.17	1.01	.814	.617	.393
D	5	↓	↓	↓	↓	↓	↓	1.65	1.53	1.40	1.24	1.07	.874	.675	.455
E	7	↓	↓	↓	↓	2.00	1.88	1.75	1.62	1.50	1.33	1.15	.955	.755	.536
F	10	↓	↓	↓	2.24	2.11	1.98	1.84	1.72	1.58	1.41	1.23	1.03	.828	.611
G	15	2.64	2.53	2.42	2.32	2.20	2.06	1.91	1.79	1.65	1.47	1.30	1.09	.886	.664
H	20	2.69	2.58	2.47	2.36	2.24	2.11	1.96	1.82	1.69	1.51	1.33	1.12	.917	.695
I	25	2.72	2.61	2.50	2.40	2.26	2.14	1.98	1.85	1.72	1.53	1.35	1.14	.936	.712
J	30	2.73	2.61	2.51	2.41	2.28	2.15	2.00	1.86	1.73	1.55	1.36	1.15	.946	.723
K	35	2.77	2.65	2.54	2.45	2.31	2.18	2.03	1.89	1.76	1.57	1.39	1.18	.969	.745
L	40	2.77	2.66	2.55	2.44	2.31	2.18	2.03	1.89	1.76	1.58	1.39	1.18	.971	.746
M	50	2.83	2.71	2.60	2.50	2.35	2.22	2.08	1.93	1.80	1.61	1.42	1.21	1.00	.774
N	75	2.90	2.77	2.66	2.55	2.41	2.27	2.12	1.98	1.84	1.65	1.46	1.24	1.03	.804
O	100	2.92	2.80	2.69	2.58	2.43	2.29	2.14	2.00	1.86	1.67	1.48	1.26	1.05	.819
P	150	2.96	2.84	2.73	2.61	2.47	2.33	2.18	2.03	1.89	1.70	1.51	1.29	1.07	.841
Q	200	2.97	2.85	2.73	2.62	2.47	2.33	2.18	2.04	1.89	1.70	1.51	1.29	1.07	.845
		.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00	
		Acceptable Quality Levels (tightened inspection)													

All AQL values are in percent defective.

↓ Use first sampling plan below arrow—that is, both sample size as well as *k* value. When sample size equals or exceeds lot size, every item in the lot must be inspected.

- Acceptance Sampling by Standards
  - Attributes
    - MIL STD 105E (ISO 2859)
    - Dodge-Romig Plan
  - Variables
    - MIL STD 414 (ISO 3951)
    - k-method
      - USL
        - Known SD
        - Unknown SD
      - LSL
        - Known SD
        - Unknown SD

Using predetermined plan for quality control