Chapter 3

Attributes Sampling Plans

Sampling Plans

- *Types of Sampling Plans:*
  1. *Single-sampling plan*
  2. *Double-sampling plan*
  3. *Multiple-sampling plan*

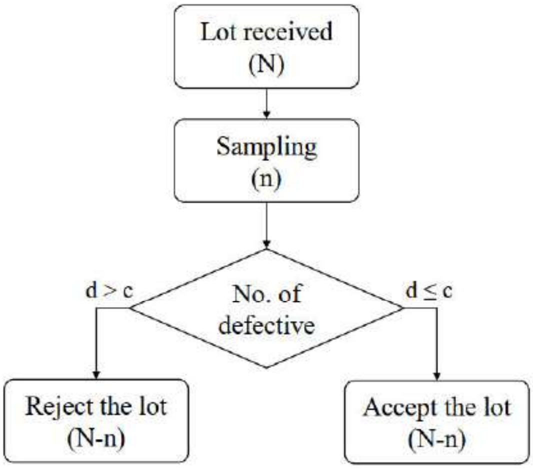
Single-sampling plan

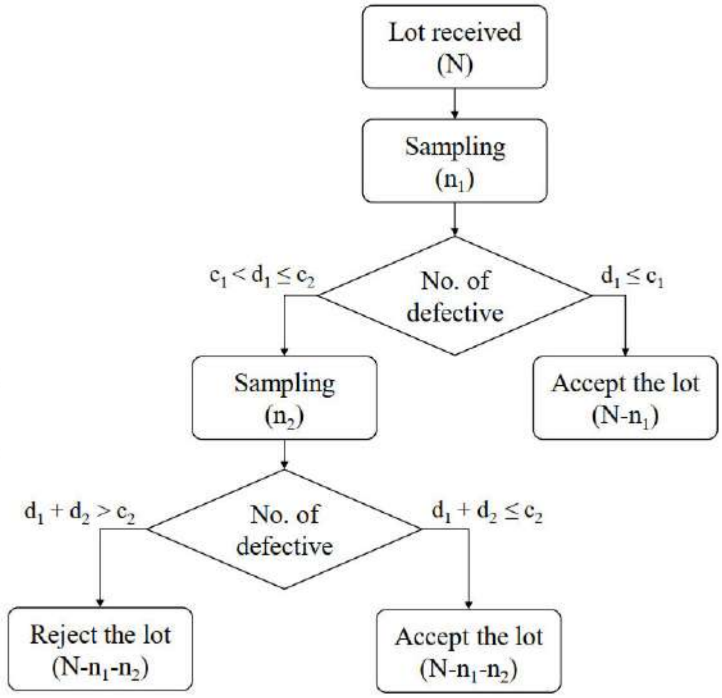
- *A lot-sentencing procedure in which **one sample of n units** is selected at random from the lot, and the disposition of the lot is determined based on the information contained in that sample*

- *For example ...*
- *a single-sampling plan for attributes would consist of a sample size n and an acceptance number c*
- *The **procedure** would operate as follows:*
    1. *Select n items at random from the lot*
    2. *If there are **c or fewer defectives** in the sample, **accept** the lot, and if there are **more than c defective** items in the sample, **reject** the lot*

Double-sampling plans

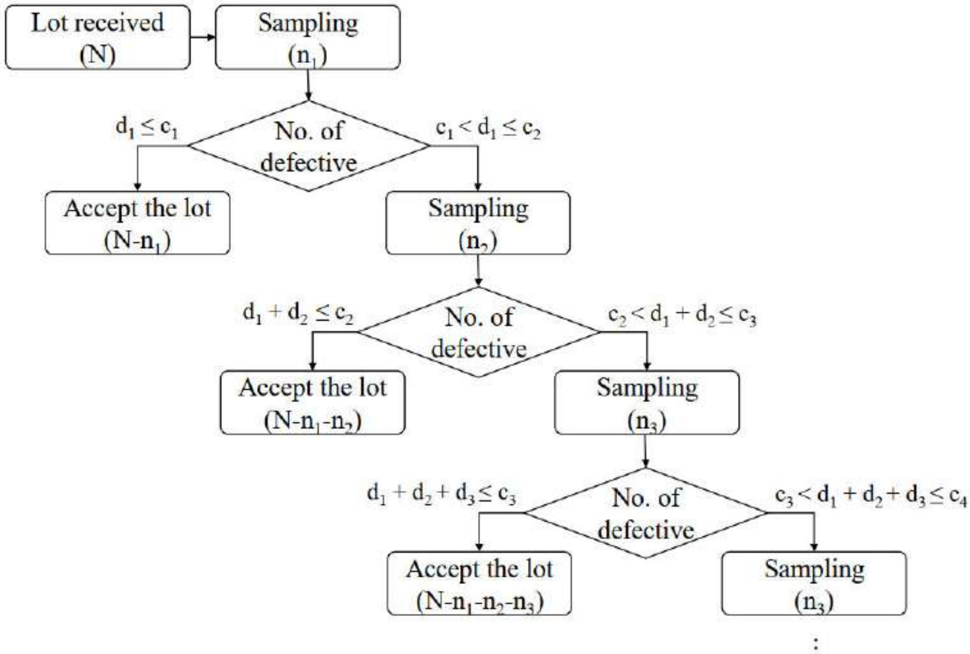
- *Somewhat more complicated!*
- *Following an initial sample, a decision based on the information in that sample is made either to*
    1. *accept the lot*
    2. *reject the lot*
    3. *take a second sample*
- *If the second sample is taken, the information from both the first and second sample is combined in order to reach a decision whether to accept or reject the lot.*





Multiple-sampling plan

- *An extension of the double-sampling concept*
- *More than two samples may be required in order to reach a decision regarding the disposition of the lot*
- *Sample sizes in multiple sampling are usually smaller than they are in either single or double sampling*
- *The ultimate extension of multiple sampling is* ***sequential sampling,***
  - *The units are selected from the lot one at a time, and following inspection of each unit, a decision is made either to accept the lot, reject the lot, or select another unit*



Single-Sampling Plans for Attributes

- *A single-sampling plan is defined by the* ***sample size n*** *and the* ***acceptance number c***
- *Suppose that a lot of size N has been submitted for inspection*
- *If the lot size is N = 10,000, then the sampling plan*

$$n = 89$$
$$c = 2$$

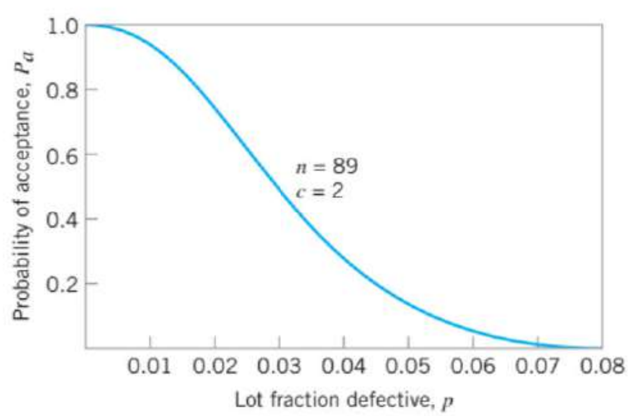
- *Means that from a lot of size 10,000 a random sample of n = 89 units is inspected and the number of nonconforming or defective items d observed*
- *If the number of observed defectives d is less than or equal to c = 2, the lot will be accepted. If the number of observed defectives d is greater than 2, the lot will be rejected.*

OC Curve in Single-Sampling Plans

- *An important measure of the performance of an acceptance-sampling plan is the Operating Characteristic (OC) curve*
- *This curve plots* ***the probability of accepting*** *the lot versus the* ***lot fraction defective***
- *It shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected*
- *The OC curve shows the* ***discriminatory power*** *of the sampling plan*

OC Curve in Single-Sampling Plans

- *The OC curve of the sampling plan n = 89, c = 2 is shown in the following*



OC Curve in Single-Sampling Plans

- *Suppose that the lot size N is large (theoretically infinite)*
- *Under this condition,*
  - *the distribution of the number of defectives d in a random sample of n items is binomial with parameters n and p,*
  - *where p is the fraction of defective items in the lot*
- *The probability of observing exactly d defectives is*

$$P\{d \text{ defectives}\} = f(d) = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d} \qquad (15.1)$$

OC Curve in Single-Sampling Plans

- *The probability of acceptance is simply the probability that d is less than or equal to c, or*

$$P_a = P\{d \leq c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d} \qquad (15.2)$$

- *For example, if the lot fraction defective is p = 0.01, n = 89, and c = 2, calculate the probability of acceptance.*

OC Curve in Single-Sampling Plans

- *So, if the lot fraction defective is p = 0.01, n = 89, and c = 2, then*

$$P_a = P\{d \le 2\} = \sum_{d=0}^{2} \frac{89!}{d!(89-d)!}(0.01)^d (0.99)^{89-d}$$
$$= \frac{89!}{0!89!}(0.01)^0(0.99)^{89} + \frac{89!}{1!88!}(0.01)^1(0.99)^{88} + \frac{89!}{2!(87)!}(0.01)^2(0.99)^{87}$$
$$= 0.9397$$

OC Curve in Single-Sampling Plans

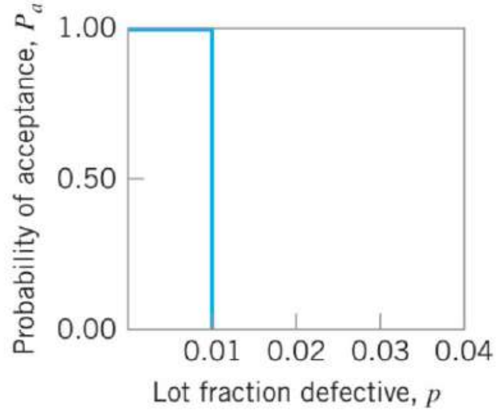
- *The OC curve shows the* ***discriminatory power*** *of the sampling plan*
- *For example, in the sampling plan n = 89, c = 2, if the lots are 2% defective, the probability of acceptance is approximately 0.74.*
- *This means that if 100 lots from a process that manufactures 2% defective product are submitted to this sampling plan, we will expect to accept 74 of the lots and reject 26 of them.*

Probabilities of Acceptance for the Single-Sampling Plan $n = 89$, $c = 2$

Fraction Defective, p	Probability of Acceptance, P_a
0.005	0.9897
0.010	0.9397
0.020	0.7366
0.030	0.4985
0.040	0.3042
0.050	0.1721
0.060	0.0919
0.070	0.0468
0.080	0.0230
0.090	0.0109

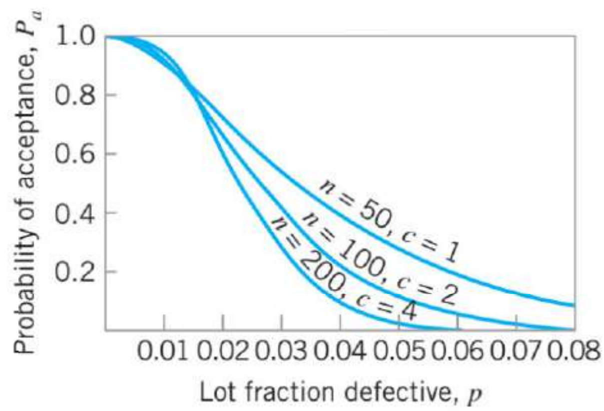
Effect of n and c on OC Curves

- *A sampling plan that discriminated perfectly between good and bad lots would have an OC curve as the figure*
- *If such a sampling plan could be employed, all lots of "bad" quality would be rejected, and all lots of "good" quality would be accepted*
- *In theory, it could be realized by 100% inspection*



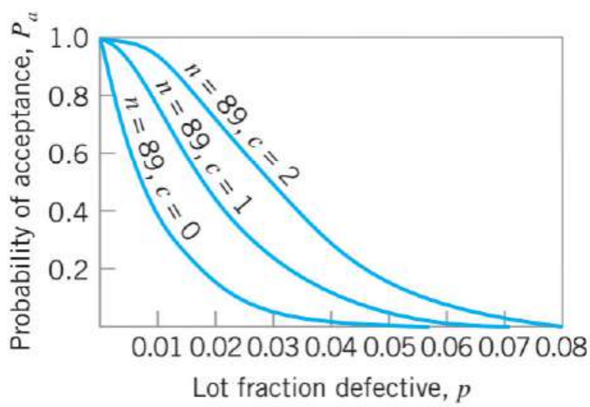
Effect of n and c on OC Curves

- *The ideal OC curve shape can be approached, however, by increasing the sample size*
- *The greater is the slope of the OC curve, the greater is the discriminatory power*



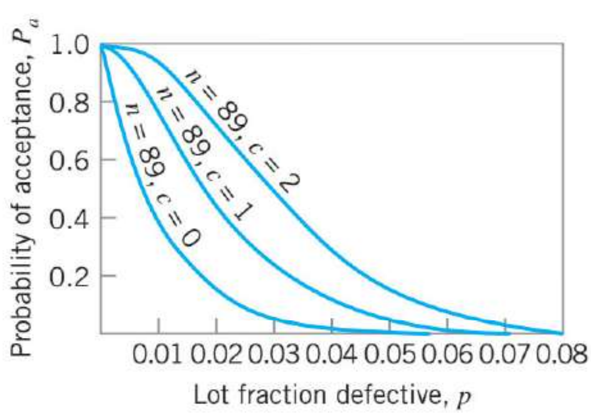
Effect of n and c on OC Curves

- *Generally, changing the acceptance number does not dramatically change the slope of the OC curve*
- *As the acceptance number is decreased, the OC curve is shifted to the left*
- *Plans with smaller values of c provide discrimination at lower levels of lot fraction defective than do plans with larger values of c*



Effect of n and c on OC Curves

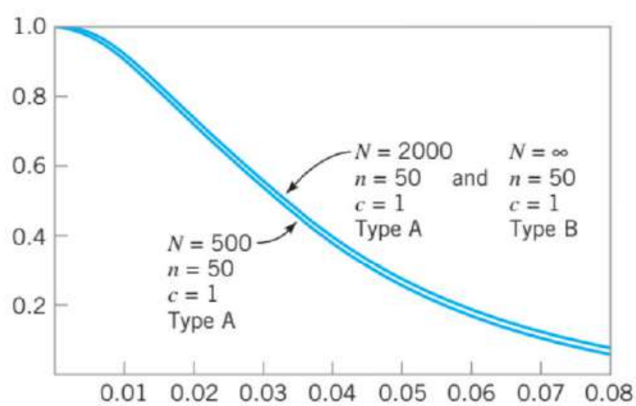
- *Plans with zero acceptance numbers are often undesirable.*
- *However, in that their OC curves are **convex** throughout.*
- *This means that the probability of lot acceptance begins to **drop very rapidly** as the lot fraction defective becomes greater than zero.*
- *This is often unfair to the producer.*



Type-A and Type-B OC Curves

- *In the construction of the OC curve it was assumed that the samples came from* ***a large lot*** *or that we were sampling from* ***a stream of lots*** *selected at random from a process.*
- *In this situation, the binomial distribution is the exact probability distribution for calculating the probability of lot acceptance.*
- *Such an OC curve is referred to as a* ***type-B OC*** *curve.*
- *The* ***type-A OC*** *curve is used to calculate probabilities of acceptance for an isolated lot of finite size.*
- *Suppose that the lot size is N, the sample size is n, and the acceptance number is c.*
- *The exact sampling distribution of the number of defective items in the sample is the hypergeometric distribution.*

Type-A and Type-B OC Curves



Specific Points on the OC Curve

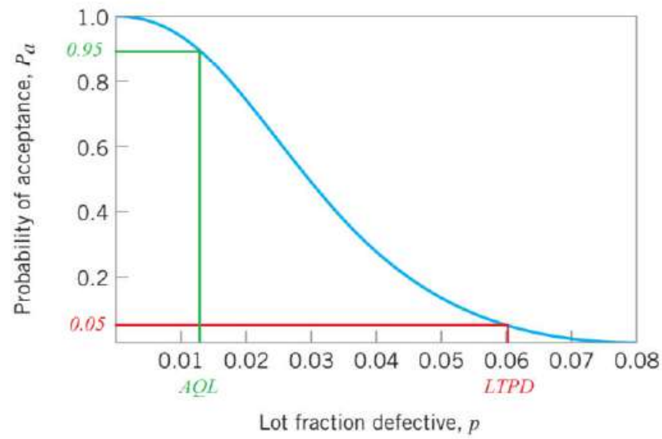
- *A consumer often establishes a sampling plan for a continuing supply of components or raw material with reference to an **acceptable quality level (AQL)**.*
- *The AQL represents the poorest level of quality for the supplier's process that the consumer would consider to be acceptable as a process average.*
- *Note that the AQL is a property of the supplier's process; it is not a property of the sampling plan*

Specific Points on the OC Curve

- *The consumer will also be interested in the other end of the OC curve—that is, in the protection that is obtained for individual lots of poor quality*
- *In such a situation, the consumer may establish a **lot tolerance percent defective (LTPD)***
- *The LTPD is the poorest level of quality that the consumer is willing to accept in an individual lot*
- *Note that the lot tolerance percent defective is not a characteristic of the sampling plan, but is a level of lot quality specified by the consumer*
- *Alternate names for the LTPD are the **rejectable quality level (RQL)** and the **limiting quality level (LQL)***

AQL vs LTPD

- *AQL and LTPD are two terms used in SPC to define* ***the acceptable level of defects in a production lot***



AQL vs LTPD

- *AQL and LTPD are two terms used in SPC to define* ***the α and β risks.***

	AQL	***LTPD***
Accepted by Consumer	*OK* *(1-α)* *95%*	*Consumer Risk* *(β)* *5%*
Rejected by Consumer	*Producer Risk* *(α)* *5%*	*OK* *(1-β)* *95%*

Designing a Single-Sampling Plan

- *A common approach to the design of an acceptance-sampling plan is to require that the OC curve pass through two designated points*
- *Suppose that we wish to construct a sampling plan such that the probability of acceptance is $1-\alpha$ for lots with fraction defective p_1, and the probability of acceptance is β for lots with fraction defective p_2*
- *Assuming that binomial sampling (with type-B OC curves) is appropriate, we see that the sample size n and acceptance number c are the solution to*

$$1-\alpha = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_1^d (1-p_1)^{n-d}$$

$$\beta = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_2^d (1-p_2)^{n-d} \qquad (15.3)$$

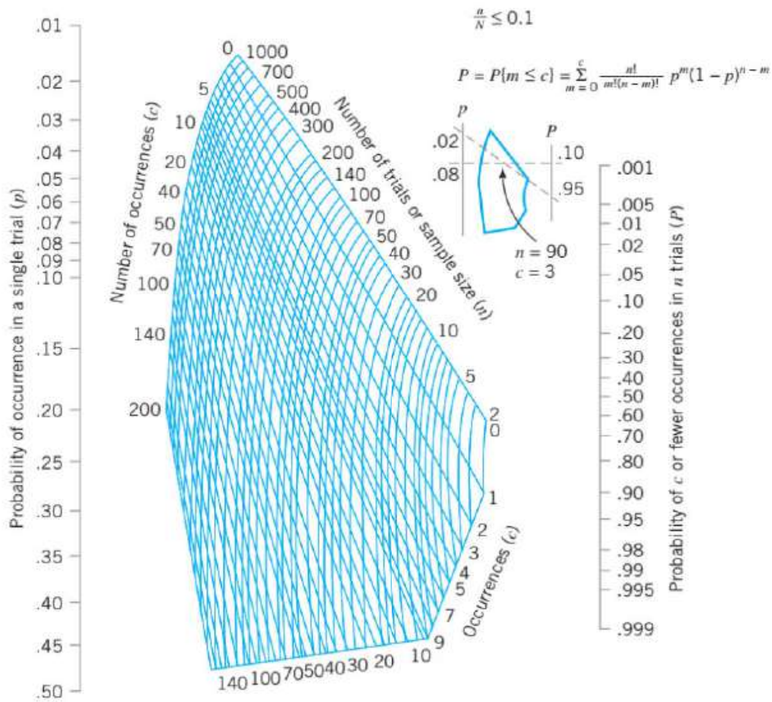
Designing a Single-Sampling Plan

- *The two simultaneous equations in Equation 15.3 are nonlinear, and there is no simple, direct solution.*

$$1-\alpha = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_1^d (1-p_1)^{n-d}$$

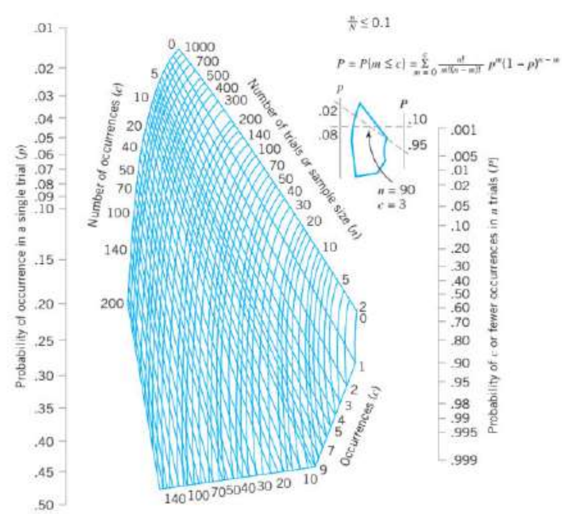
$$\beta = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_2^d (1-p_2)^{n-d} \qquad (15.3)$$

- *The nomograph in the next figure can be used for solving these equations.*

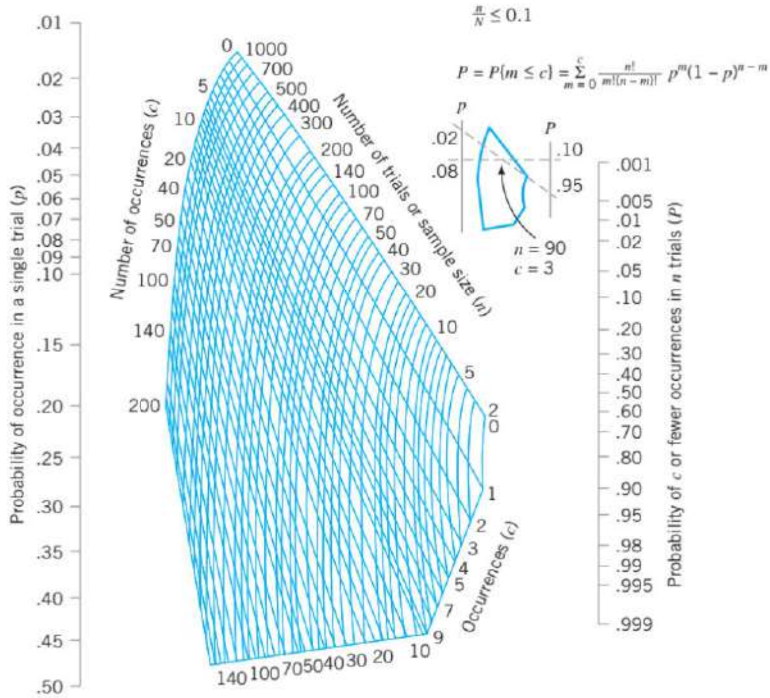


Designing a Single-Sampling Plan

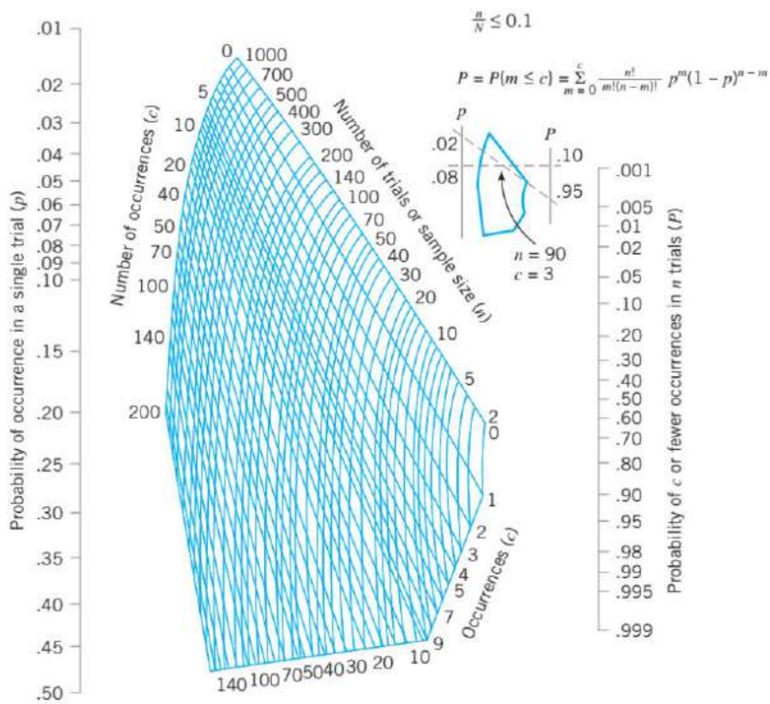
- *The procedure is to draw two lines on the nomograph*
- *One connecting p_1 and $1-\alpha$, and the other connecting p_2 and β*
- *The intersection of these two lines gives the region of the nomograph in which the desired sampling plan is located.*



- *For example, suppose we wish to construct a sampling plan for which p_1=0.01, α=0.05, p_2=0.06, and β=0.10.*



- *Locating the intersection of the lines connecting*
  - *p_1=0.01, 1−α=0.95*
  - *p_2=0.06, β=0.10*
- *on the nomograph indicates that the plan n = 89, c = 2 is very close to passing through these two points on the OC curve*

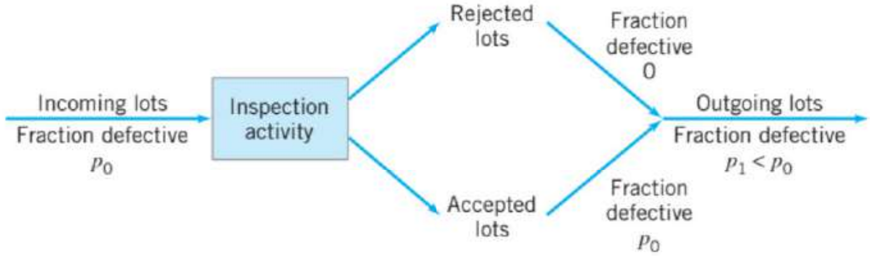


Screening or Rectifying Inspection

- *Acceptance-sampling programs usually require corrective action when lots are rejected*
- *This generally takes the form of **100% inspection** or **screening** of rejected lots*
- *Such sampling programs are called **rectifying inspection programs** because the inspection activity affects the final quality of the outgoing product*

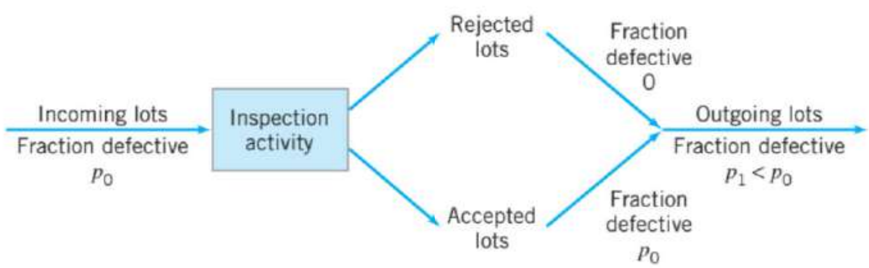
Screening or Rectifying Inspection

- *A rectifying inspection programs will work as follow:*



Screening or Rectifying Inspection

- *Average Outgoing Quality (AOQ) is widely used for the evaluation of a rectifying sampling plan*
- *The average outgoing quality is the quality in the lot that results from the application of rectifying inspection*



Screening or Rectifying Inspection

- *Assume that the lot size is N and that all discovered defectives are replaced with good units.*
- *Then in lots of size N, we have*
  1. *n items in the sample that, after inspection, contain no defectives, because all discovered defectives are replaced*
  2. *$N - n$ items that, if the lot is rejected, also contain no defectives*
  3. *$N - n$ items that, if the lot is accepted, contain $p(N - n)$ defectives*

Screening or Rectifying Inspection

- *Thus, lots in the outgoing stage of inspection have an expected number of defective units equal to* $\boldsymbol{P_a\, p(N - n)}$, *which we may express as an average fraction defective, called the* ***average outgoing quality*** *or*

$$\text{AOQ} = \frac{P_a p(N-n)}{N} \qquad (15.4)$$

Screening or Rectifying Inspection

- *For example, suppose that*
  - $N = 10{,}000$
  - $n = 89$
  - $c = 2$
  - *the quality of the lots is* $p = 0.01$
  - *the probability of acceptance* $P_a = 0.9397$
- *Calculate the AOQ …*

Screening or Rectifying Inspection

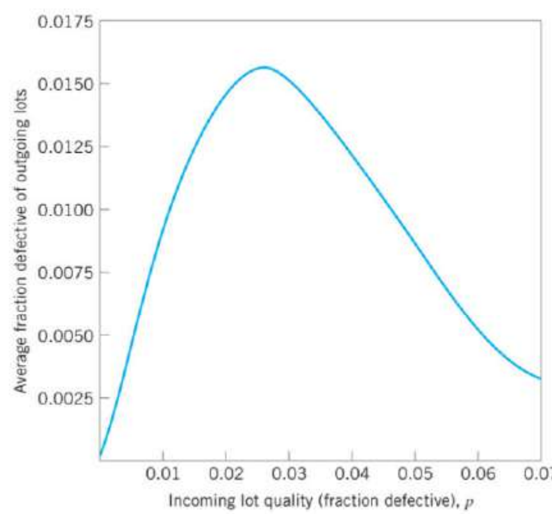
- *So,*

$$P_a = P\{d \le 2\} = \sum_{d=0}^{2} \frac{89!}{d!(89-d)!}(0.01)^d (0.99)^{89-d}$$
$$= \frac{89!}{0!89!}(0.01)^0(0.99)^{89} + \frac{89!}{1!88!}(0.01)^1(0.99)^{88} + \frac{89!}{2!(87)!}(0.01)^2(0.99)^{87}$$
$$= 0.9397$$

$$\text{AOQ} = \frac{P_a p(N-n)}{N}$$
$$= \frac{(0.9397)(0.01)(10,000-89)}{10,000}$$
$$= 0.0093$$ *That is, the average outgoing quality is 0.93% defective*

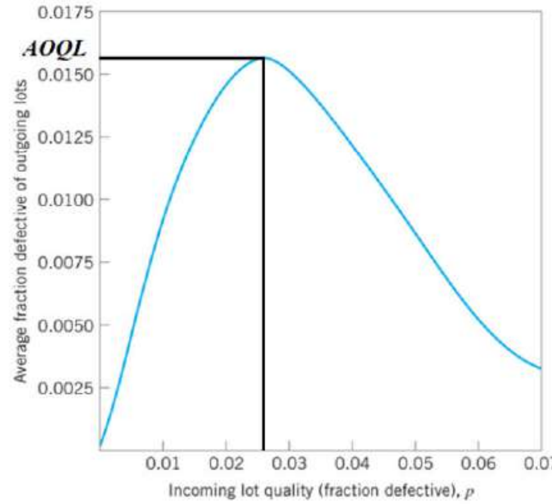
Screening or Rectifying Inspection

- *Average outgoing quality curve for n = 89, c = 2*
- *This curve we note that when the incoming quality is very good, the average outgoing quality is also very good*
- *In contrast, when the incoming lot quality is very bad, most of the lots are rejected and screened, which leads to a very good level of quality in the outgoing lots*



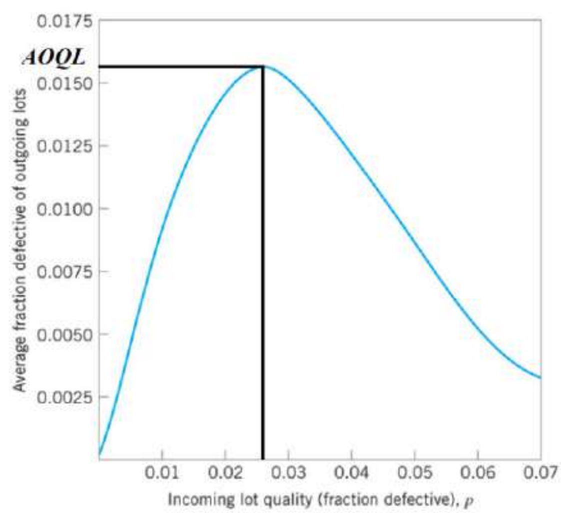
Screening or Rectifying Inspection

- *The maximum ordinate on the AOQ curve represents the worst possible average quality that would result from the rectifying inspection program, and this point is called the* ***Average Outgoing Quality Limit (AOQL)***



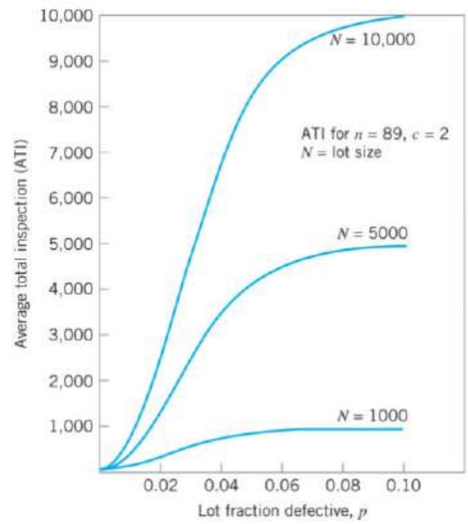
Screening or Rectifying Inspection

- *The AOQL is seen to be approximately 0.0155*
- *That is, no matter how bad the fraction defective is in the incoming lots, the outgoing lots will never have a worse quality level on the average than 1.55% defective*



Screening or Rectifying Inspection

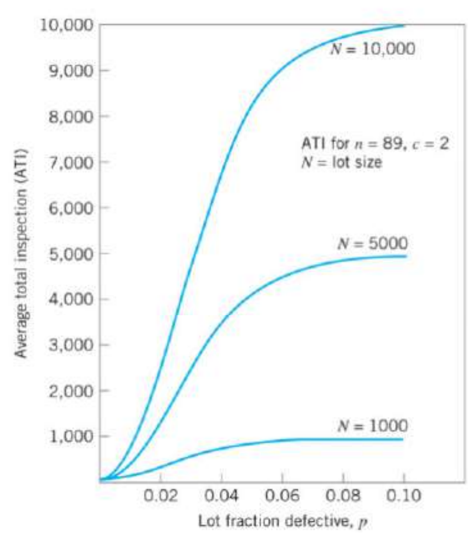
- *Another important measure relative to rectifying inspection is the total amount of inspection required by the sampling program.*
- *If the lots contain no defective items, no lots will be rejected, and the amount of inspection per lot will be the sample size n.*
- *If the items are all defective, every lot will be submitted to 100% inspection, and the amount of inspection per lot will be the lot size N*



Screening or Rectifying Inspection

- *If the lot quality is $0 < p < 1$, the average amount of inspection per lot will vary between the sample size n and the lot size N.*
- *If the lot is of quality p and the probability of lot acceptance is P_a, then the average total inspection per lot will be*

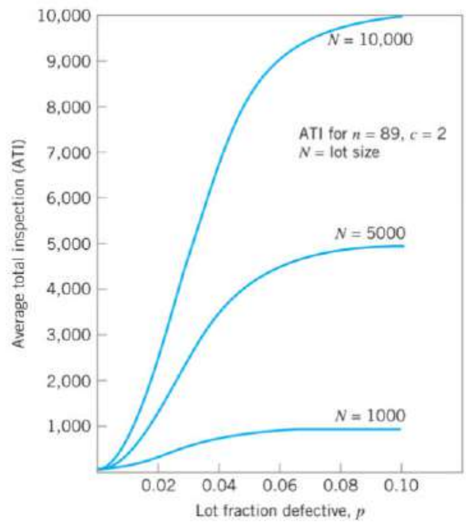
$$\text{ATI} = n + (1 - P_a)(N - n)$$



Screening or Rectifying Inspection

- *Consider our previous example with N = 10,000, n = 89, c = 2, and p = 0.01.*
- *Since Pa = 0.9397, we have*

$$\begin{aligned}\text{ATI} &= n + (1 - P_a)(N - n) \\ &= 89 + (1 - 0.9397)(10{,}000 - 89) \\ &= 687\end{aligned}$$



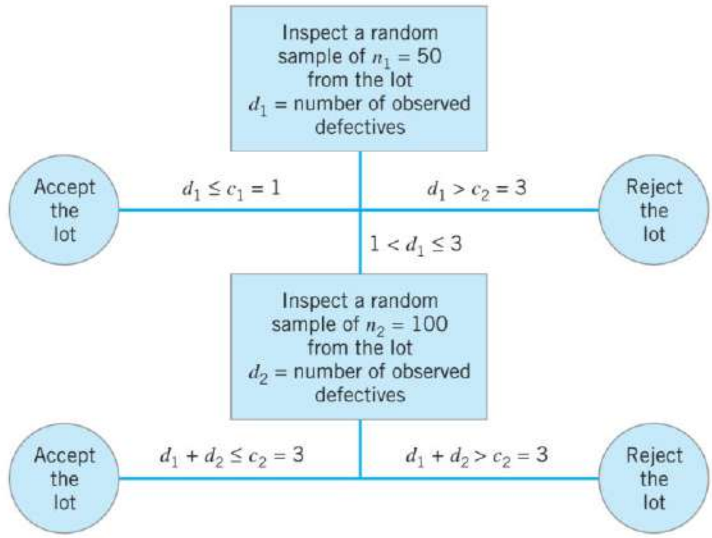
Double-Sampling Plans

- *A double-sampling plan is a procedure in which, under certain circumstances, a second sample is required before the lot can be sentenced.*
- *A double-sampling plan is defined by four parameters*

n_1 = sample size on the first sample
c_1 = acceptance number of the first sample
n_2 = sample size on the second sample
c_2 = acceptance number for both sample

Double-Sampling Plans

- *As example operation of double sampling plan, n_1=50, c_1=1, n_2=100, c_2=3.*

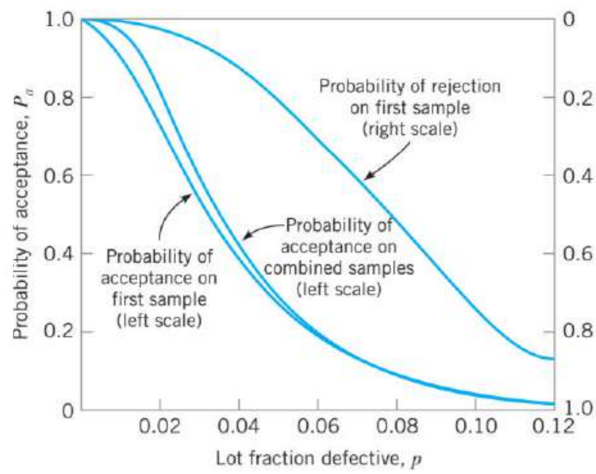


OC Curve for Double-Sampling Plans

- *The performance of a double-sampling plan can be conveniently summarized by means of its OC curve*
- *A double-sampling plan has a **primary** OC curve that gives the probability of acceptance as a function of lot or process quality.*
- *It also has **supplementary** OC curves that show the probability of lot acceptance and rejection on the first sample.*
- *The OC curve for the probability of rejection on the first sample is simply the OC curve for the single-sampling plan $n = n_1$ and $c = c_2$.*

OC Curve for Double-Sampling Plans

- *Primary and supplementary OC curves for the plan $n_1=50$, $c_1=1$, $n_2=100$, $c_2=3$ are:*
- *In the next step, we want to calculate them ...*



OC Curve for Double-Sampling Plans

- *If P_a denotes the probability of acceptance on the combined samples, and*
  - P_a^I *denote the probability of acceptance on the first sample*
  - P_a^{II} *denote the probability of acceptance on the second sample*
- *then,*

$$P_a = P_a^{\mathrm{I}} + P_a^{\mathrm{II}}$$

OC Curve for Double-Sampling Plans

- P_a^I *is just the probability that we will observe* $d_1 \leq c_1 = 1$ *defectives out of a random sample of* $n_1 = 50$ *items. Thus*

$$P_a^{\mathrm{I}} = \sum_{d_1=0}^{1} \frac{50!}{d_1!(50-d_1)!} p^{d_1} (1-p)^{50-d_1}$$

OC Curve for Double-Sampling Plans

- P_a^{II} *is the probability of acceptance on the second sample, we must list the number of ways the second sample can be obtained. A second sample is drawn only if there are two or three defectives on the first sample—that is, if* $c_1 < d_1 \leq c_2$.

1. $d_1 = 2$ *and* $d_2 = 0$ *or 1,* $P\{d_1 = 2, d_2 \leq 1\} = P\{d_1 = 2\} \cdot P\{d_2 \leq 1\}$
2. $d_1 = 3$ *and* $d_2 = 0$, $P\{d_1 = 3, d_2 = 0\} = P\{d_1 = 3\} \cdot P\{d_2 = 0\}$

$$P_a^{\mathrm{II}} = P\{d_1 = 2, d_2 \leq 1\} + P\{d_1 = 3, d_2 = 0\}$$

OC Curve for Double-Sampling Plans

- *If p = 0.05 is the fraction defective in the incoming lot, then*

$$P_a^{\mathrm{I}} = \sum_{d_1=0}^{1} \frac{50!}{d_1!(50-d_1)!}(0.05)^{d_1}(0.95)^{50-d_1} = 0.279$$

$$\begin{aligned} P\{d_1 = 2, d_2 \le 1\} &= P\{d_1 = 2\} \cdot P\{d_2 \le 1\} \\ &= \frac{50!}{2!48!}(0.05)^2(0.95)^{48} \sum_{d_2=0}^{1} \frac{100!}{d_2!(100-d_2)!}(0.05)^{d_2}(0.95)^{100-d_2} \\ &= (0.261)(0.037) = 0.0097 \end{aligned}$$

$$\begin{aligned} P\{d_1 = 3, d_2 = 0\} &= P\{d_1 = 3\} \cdot P\{d_2 = 0\} \\ &= \frac{50!}{3!(47)!}(0.05)^3(0.95)^{47} \frac{100!}{0!100!}(0.05)^0(0.95)^{100} \\ &= (0.220)(0.0059) = 0.001 \end{aligned}$$

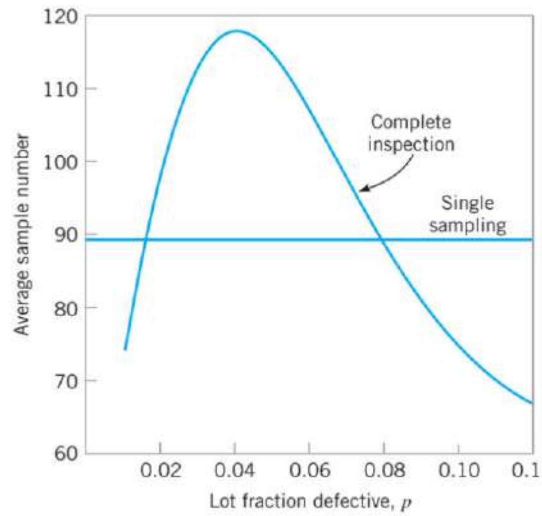
OC Curve for Double-Sampling Plans

- *The probability of acceptance of a lot that has fraction defective p = 0.05 is therefore*

$$\begin{aligned} P_a &= P_a^{\mathrm{I}} + P_a^{\mathrm{II}} \\ &= 0.279 + 0.0107 = 0.2897 \end{aligned}$$

The Average Sample Number Curve

- *In single sampling, the size of the sample inspected from the lot is always constant.*
- *Whereas in double sampling the size of the sample selected depends on whether or not the second sample is necessary.*



The Average Sample Number Curve

- *The formula for the average sample number in double sampling is:*

$$\begin{aligned} \text{ASN} &= n_1 P_I + (n_1 + n_2)(1 - P_I) \\ &= n_1 + n_2(1 - P_I) \end{aligned}$$

- *where P_I is the probability of making a lot-dispositioning decision on the first sample. This is*

$$P_I = P\{\text{lot is accepted on the first sample}\} + P\{\text{lot is rejected on the first sample}\}$$

Curtailment

- ***In practice**, inspection of the second sample is usually terminated and the lot rejected as soon as the number of observed defective items in the combined sample exceeds the acceptance number c.*
  - *For c_2, referred as curtailment of the second sample*
  - *For c_1, referred as curtailment of the first sample*
  - *For c, referred as curtailment of single sampling*

- *The use of curtailed inspection lowers the average sample number required in double sampling*
- *It is not recommended!!*

ASN Curve with Curtailment

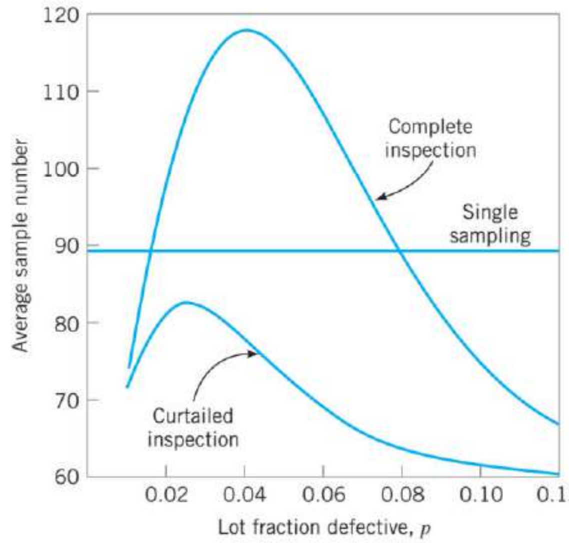
- *The ASN curve formula for a double-sampling plan with curtailment on the second sample is*

$$\text{ASN} = n_1 + \sum_{j=c_1+1}^{c_2} P(n_1, j)\left[n_2 P_L(n_2, c_2 - j) + \frac{c_2 - j + 1}{p} P_M(n_2 + 1, c_2 - j + 2)\right]$$

  - *$P(n_1, j)$ is the probability of observing j defectives in a sample of size n_1*
  - *$P_L(n_2, c_2 - j)$ is the probability of observing $c_2 - j$ or fewer defectives in a sample of size n_2*
  - *$P_M(n_2 + 1, c_2 - j + 2)$ is the probability of observing $c_2 - j + 2$ defectives in a sample of size $n_2 + 1$*

ASN Curve with Curtailment

- *The figure compares the average sample number curves for complete and curtailed inspection for the double-sampling plan $n_1=60$, $c_1=2$, $n_2=120$, $c_3=3$, and the average sample number that would be used in single-sampling with $n = 89$, $c = 2$*



Designing Double-Sampling Plans

- *Let $(p_1, 1 - \alpha)$ and (p_2, β) be the two points of interest on the OC curve.*
- *If, in addition, we impose another relationship on the parameters of the sampling plan, then a simple procedure can be used to obtain such plans.*
- *The most common constraint is to require that n_2 is a multiple of n_1.*
- *Refer to:*

- *Duncan, A. J. (1986). Quality Control and Industrial Statistics, 5th ed.,*

Rectifying Inspection in Double-Sampling Plans

- *When rectifying inspection is performed with double sampling, the AOQ curve is given by*

$$\text{AOQ} = \frac{\left[P_a^{\text{I}}(N - n_1) + P_a^{\text{II}}(N - n_1 - n_2)\right]p}{N}$$

- *The average total inspection curve is given by*

$$\text{ATI} = n_1 P_a^{\text{I}} + (n_1 + n_2)P_a^{\text{II}} + N(1 - P_a)$$

- *Remember that* $P_a = P_a^I + P_a^{II}$ *is the probability of final lot acceptance and that the acceptance probabilities depend on the level of lot or process quality p.*

Sequential Sampling Plans

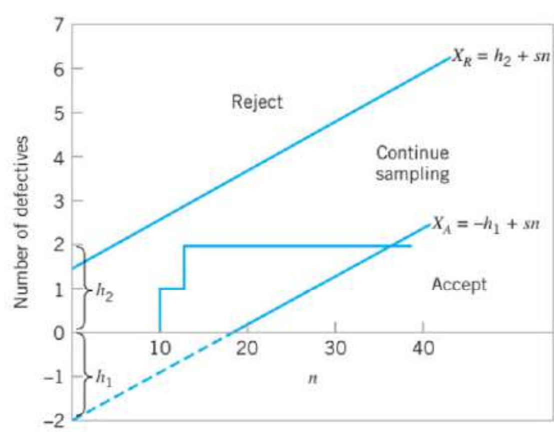
- *Sequential sampling is an extension of the double-sampling and multiple-sampling concept.*
- *In sequential sampling, we take a sequence of samples from the lot and allow the number of samples to be determined entirely by the results of the sampling process.*
- *Sequential sampling can theoretically continue indefinitely, until the lot is inspected 100%.*
- ***In practice,*** *sequential sampling plans are usually truncated after three times inspection.*

Sequential Sampling Plans

- *Types of sequential sampling:*
  - *Group sequential sampling: if the sample size selected at each stage is greater than one*
  - *Item-by-item sequential sampling: if the sample size inspected at each stage is one*
- *Item-by-item sequential sampling is based on the sequential probability ratio test (SPRT), developed by Wald (1947)*

Sequential Sampling Plans

- *The cumulative observed number of defectives is plotted on the chart.*
- *The abscissa is the total number of items selected up to that time,*
- *The ordinate is the total number of observed defectives.*
- *If the plotted points stay within the boundaries of the acceptance and rejection lines, another sample must be drawn.*



Sequential Sampling Plans

- *The equations for the two limit lines for specified values of p_1, $1-\alpha$, p_2, and β are:*

$$X_A = -h_1 + sn \qquad \text{(acceptance line)}$$

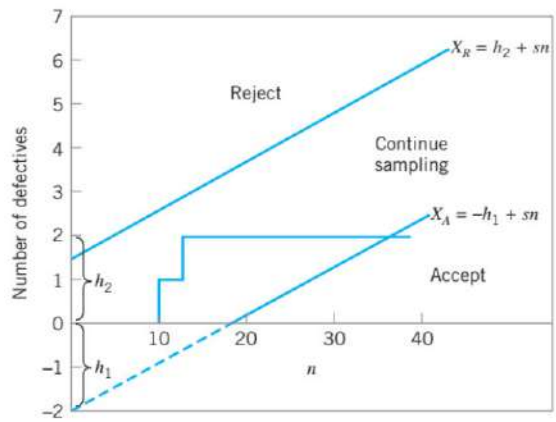
$$X_R = h_2 + sn \qquad \text{(rejection line)}$$

$$h_1 = \left(\log \frac{1-\alpha}{\beta}\right) \Big/ k$$

$$h_2 = \left(\log \frac{1-\beta}{\alpha}\right) \Big/ k$$

$$k = \log \frac{p_2(1-p_1)}{p_1(1-p_2)}$$

$$s = \log\left[(1-p_1)/(1-p_2)\right]/k$$



Sequential Sampling Plans

- *Suppose we wish to find a sequential-sampling plan for which p_1=0.01, α=0.05, p_2=0.06, and β=0.10*

$$k = \log \frac{p_2(1-p_1)}{p_1(1-p_2)} = \log \frac{(0.06)(0.99)}{(0.01)(0.94)} = 0.80066$$

$$s = \log\left[(1-p_1)/(1-p_2)\right]/k = \left[\log(0.99/0.94)\right]/0.80066 = 0.028$$

$$h_1 = \left(\log \frac{1-\alpha}{\beta}\right) \Big/ k = \left(\log \frac{0.95}{0.10}\right) \Big/ 0.80066 = 1.22$$

$$h_2 = \left(\log \frac{1-\beta}{\alpha}\right) \Big/ k = \left(\log \frac{0.90}{0.05}\right) \Big/ 0.80066 = 1.57$$

Sequential Sampling Plans

- *Suppose we wish to find a sequential-sampling plan for which p_1=0.01, α=0.05, p_2=0.06, and β=0.10*
- *Therefore, the limit lines are*

$$X_A = -1.22 + 0.028n \quad \text{(accept)}$$

$$X_R = 1.57 + 0.028n \quad \text{(reject)}$$

ASN Curve in Sequential Sampling

- *The average sample number taken under sequential-sampling is*

$$\text{ASN} = P_a\left(\frac{A}{C}\right) + (1 - P_a)\frac{B}{C}$$

$$A = \log\frac{\beta}{1-\alpha}$$

$$B = \log\frac{1-\beta}{\alpha}$$

$$C = p\log\left(\frac{p_2}{p_1}\right) + (1-p)\log\left(\frac{1-p_2}{1-p_1}\right)$$

Rectifying Inspection in Sequential Sampling

- *The average outgoing quality (AOQ) for sequential sampling is given approximately by*

$$\text{AOQ} \simeq P_a p$$

- *The average total inspection is*

$$\text{ATI} = P_a\left(\frac{A}{C}\right) + (1 - P_a)N$$

Attribute Acceptance Sampling

- Single plan
  - Parameters: n , c
  - ASN = n
  - Rectifying
    - $\text{AOQ} = \frac{P_a p(N-n)}{N}$
    - $\text{ATI} = n + (1-P_a)(N-n)$
- Double plan
  - Parameters: n_1 , c_1 , n_2 , c_2
  - $\text{ASN} = n_1 + n_2(1 - P_1)$
  - Rectifying
    - $\text{AOQ} = \frac{\left[P_a^{\text{I}}(N-n_1) + P_a^{\text{II}}(N-n_1-n_2)\right]p}{N}$
    - $\text{ATI} = n_1 P_a^{\text{I}} + (n_1+n_2)P_a^{\text{II}} + N(1-P_a)$
- Sequential plan
  - Parameters: n_i , c_i , …
  - $\text{ASN} = P_a\left(\frac{A}{C}\right) + (1-P_a)\frac{B}{C}$
  - Rectifying
    - $\text{AOQ} \simeq P_a p$
    - $\text{ATI} = P_a\left(\frac{A}{C}\right) + (1-P_a)N$

Attribute Acceptance Sampling → Designing our intended plan for quality control

In all case, we use Duncan (1986) method for plotting the OC curve using $(p_1, 1-\alpha)$ and (p_2, β) on nomogram

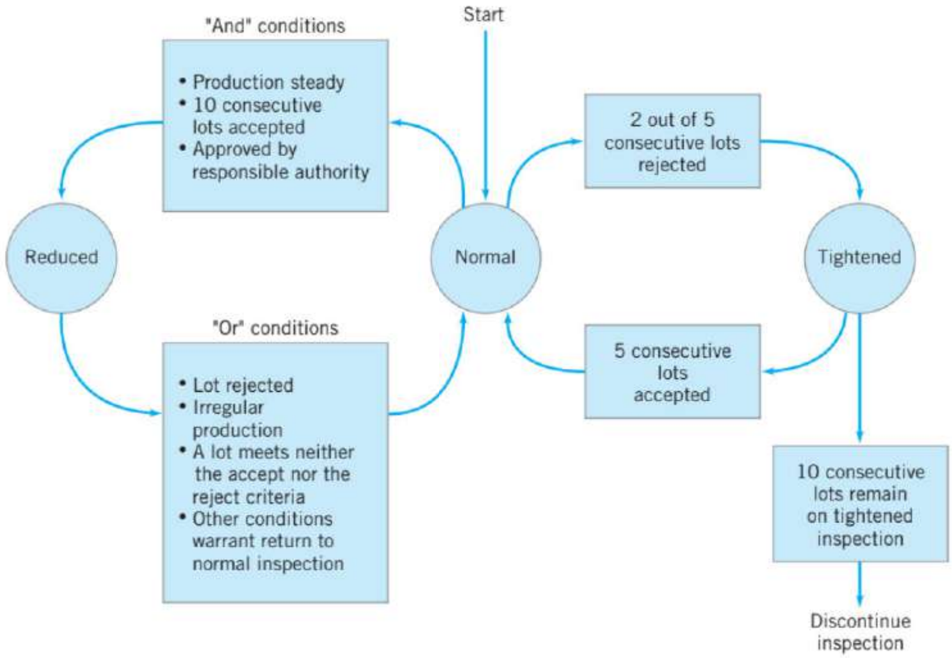
Military Standard 105E

- *Standard sampling procedures for inspection by attributes were developed during World War II*
- *MIL STD 105E is the most widely used acceptance-sampling system for attributes in the world today.*
- *The original version of the standard, MIL STD 105A, was issued in 1950*
- *The latest version is MIL STD 105E, which was issued in 1989.*
- *The standard was also adopted by the International Organization for Standardization as ISO 2859*

Military Standard 105E

- *The standard provides for three types of sampling:*
  - *single sampling*
  - *double sampling*
  - *multiple sampling*
- *For each type of sampling plan, a provision is made for either*
  - *normal inspection: used at the start of the inspection activity*
  - *tightened inspection: used when the supplier's recent quality history has deteriorated*
  - *reduced inspection: used when the supplier's recent quality history has been exceptionally good*

Military Standard 105E



Procedure for using MIL STD 105E

1. *Choose the AQL.*
2. *Choose the inspection level.*
3. *Determine the lot size.*
4. *Find the appropriate sample size code letter.*
5. *Determine the appropriate type of sampling plan to use.*
6. *Enter the appropriate table to find the type of plan to be used.*
7. *Determine the corresponding normal and reduced inspection plans to be used when required.*

TABLE 15.4

Sample Size Code Letters (MIL STD 105E, Table 1)

	Special Inspection Levels				General Inspection Levels		
Lot or Batch Size	**S-1**	**S-2**	**S-3**	**S-4**	**I**	**II**	**III**
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1,200	C	C	E	F	G	J	K
1,201 to 3,200	C	D	E	G	H	K	L
3,201 to 10,000	C	D	F	G	J	L	M
10,001 to 35,000	C	D	F	H	K	M	N
35,001 to 150,000	D	E	G	J	L	N	P
150,001 to 500,000	D	E	G	J	M	P	Q
500,001 and over	D	E	H	K	N	Q	R

Master Table for Normal Inspection for Single Sampling (U.S. Dept. of Defense MIL STD 105E, Table II-A)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (normal inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

TABLE 15.5

Master Table for Tightened Inspection for Single Sampling (U.S. Dept. of Defense MIL STD 105E, Table II-B)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (tightened inspection) 0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 11	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150			1 2																							

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

TABLE 15.6

Master Table for Reduced Inspection for Single Sampling (U.S. Dept. of Defense MIL STD 105E, Table II-C)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (reduced inspection)† 0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	2 4	3 5	5 6	7 8	10 11	14 15	21 22	30 31
C	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑
D	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑
E	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑	↑
F	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑
G	13	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑
H	20	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑
J	32	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	50	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	80	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	125	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	200	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	315	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	500	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	800	↑	↑	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

† = If the acceptance number has been exceeded, but the rejection number has not been reached, accept the lot, but reinstate normal inspection.

TABLE 15.7

Military Standard 105E

- *Example.*
- *Suppose that a product is submitted in lots of size N = 2,000.*
- *The acceptable quality level is 0.65%.*
- *We will use the standard to generate normal, tightened, and reduced single-sampling plans for this situation.*

Military Standard 105E

- *Answer.*
- *For lots of size 2,000 under general inspection level II,* ***Table 15.4*** *indicates that the appropriate sample size code letter is K*
- *Therefore, from* ***Table 15.5****, for single-sampling plans under normal inspection, the normal inspection plan is n=125, c=2.*
- *Table 15.6 indicates that the corresponding tightened inspection plan is n=125, c=1.*
- *Table 15.7 indicates that under reduced inspection, the sample size for this example would be n=50, the acceptance number would be c=1, and the rejection number would be r=3*

The Dodge–Romig Sampling Plans

- *H. F. Dodge and H. G. Romig (1959) developed a set of sampling inspection tables for lot-by-lot inspection of product by attributes using two types of sampling plans:*
  - *plans for lot tolerance percent defective (LTPD)*
  - *plans that provide a specified average outgoing quality limit (AOQL)*

The Dodge–Romig Sampling Plans

- *The Dodge–Romig AOQL plans are designed so that the average total inspection for a given AOQL and a specified process average p will be minimized.*
- *Similarly, the LTPD plans are designed so that the average total inspection is a minimum.*
- *This makes the Dodge–Romig plans very useful for in-plant inspection of semifinished product.*

AOQL Plans

- *Example.*
- *Suppose that we are inspecting LSI memory elements for a personal computer and that the elements are shipped in lots of size N = 5,000.*
- *The supplier's process average fallout is 1% nonconforming.*
- *We wish to find a single-sampling plan with an AOQL = 3%.*
- *From Table, we find that the plan is*

$$n = 65 \qquad c = 3$$

- *Table also indicates that the LTPD for this sampling plan is 10.3%.*

Dodge–Romig Inspection Table for Single-Sampling Plans for AOQL = 3.0%

	Process Average																	
	0–0.06%			0.07–0.60%			0.61–1.20%			1.21–1.80%			1.81–2.40%			2.41–3.00%		
Lot Size	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %
1–10	All	0	—	All	0	—	All	0	—	All	0	—	All	0	—	All	0	—
11–50	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0
51–100	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	22	1	16.4
101–200	12	0	17.0	12	0	17.0	12	0	17.0	25	1	15.1	25	1	15.1	25	1	15.1
201–300	12	0	17.0	12	0	17.0	26	1	14.6	26	1	14.6	26	1	14.6	40	2	12.8
301–400	12	0	17.1	12	0	17.1	26	1	14.7	26	1	14.7	41	2	12.7	41	2	12.7
401–500	12	0	17.2	27	1	14.1	27	1	14.1	42	2	12.4	42	2	12.4	42	2	12.4
501–600	12	0	17.3	27	1	14.2	27	1	14.2	42	2	12.4	42	2	12.4	60	3	10.8
601–800	12	0	17.3	27	1	14.2	27	1	14.2	43	2	12.1	60	3	10.9	60	3	10.9
801–1000	12	0	17.4	27	1	14.2	44	2	11.8	44	2	11.8	60	3	11.0	80	4	9.8
1,001–2,000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	80	4	9.8	100	5	9.1
2,001–3,000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	100	5	9.1	140	7	8.2
3,001–4,000	12	0	17.5	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	165	8	7.8
4,001–5,000	28	1	13.8	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	210	10	7.4
5,001–7,000	28	1	13.8	45	2	11.8	65	3	10.3	105	5	8.8	145	7	8.1	235	11	7.1
7,001–10,000	28	1	13.9	46	2	11.6	65	3	10.3	105	5	8.8	170	8	7.6	280	13	6.8
10,001–20,000	28	1	13.9	46	2	11.7	85	4	9.5	125	6	8.4	215	10	7.2	380	17	6.2
20,001–50,000	28	1	13.9	65	3	10.3	105	5	8.8	170	8	7.6	310	14	6.5	560	24	5.7
50,001–100,000	28	1	13.9	65	3	10.3	125	6	8.4	215	10	7.2	385	17	6.2	690	29	5.4

LTPD Plans

- *Example.*
- *Suppose that we are inspecting LSI memory elements for a personal computer and that the elements are shipped in lots of size N = 5,000.*
- *The supplier's process average fallout is 0.25% nonconforming.*
- *We wish to find a single-sampling plan with an AOQL = 1%.*
- *From Table, we find that the plan is*

$$n = 770 \qquad c = 4$$

- *Table also indicates that the AOQL for this sampling plan is 0.28%.*

Dodge–Romig Single-Sampling Table for Lot Tolerance Percent Defective (LTPD) = 1.0%

	Process Average																	
	0–0.01%			0.011%–0.10%			0.11–0.20%			0.21–0.30%			0.31–0.40%			0.41–0.50%		
Lot Size	*n*	*c*	AOQL %	*n*	*c*	AOQL %	*n*	*c*	AOQL %	*n*	*c*	AOQL %	*n*	*c*	AOQL %	*n*	*c*	AOQL %
1–120	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0
121–150	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06
151–200	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08
201–300	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10
301–400	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12
401–500	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13
501–600	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	305	1	0.14
601–800	200	0	0.14	200	0	0.14	200	0	0.14	330	1	0.15	330	1	0.15	330	1	0.15
801–1000	205	0	0.14	205	0	0.14	205	0	0.14	335	1	0.17	335	1	0.17	335	1	0.17
1,001–2,000	220	0	0.15	220	0	0.15	360	1	0.19	490	2	0.21	490	2	0.21	610	3	0.22
2,001–3,000	220	0	0.15	375	1	0.20	505	2	0.23	630	3	0.24	745	4	0.26	870	5	0.26
3,001–4,000	225	0	0.15	380	1	0.20	510	2	0.23	645	3	0.25	880	5	0.28	1,000	6	0.29
4,001–5,000	225	0	0.16	380	1	0.20	520	2	0.24	770	4	0.28	895	5	0.29	1,120	7	0.31
5,001–7,000	230	0	0.16	385	1	0.21	655	3	0.27	780	4	0.29	1,020	6	0.32	1,260	8	0.34
7,001–10,000	230	0	0.16	520	2	0.25	660	3	0.28	910	5	0.32	1,150	7	0.34	1,500	10	0.37
10,001–20,000	390	1	0.21	525	2	0.26	785	4	0.31	1,040	6	0.35	1,400	9	0.39	1,980	14	0.43
20,001–50,000	390	1	0.21	530	2	0.26	920	5	0.34	1,300	8	0.39	1,890	13	0.44	2,570	19	0.48
50,001–100,000	390	1	0.21	670	3	0.29	1,040	6	0.36	1,420	9	0.41	2,120	15	0.47	3,150	23	0.50